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THE CARBON MICROPHONE: AN ACCOUNT OF SOME RESEARCHES BEARING ON ITS ACTION.*

BY

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INTRODUCTION.

Few electrical devices are as widely used as the "carbon microphone" and few have given rise to as much speculation in regard to their mode of action. That the problem has proved elusive is shown by the fact that in Bell Telephone Laboratories it has been regarded as perennial. However, recent researches have thrown a considerable amount of light upon it and it therefore seems fitting to bring before you this evening a brief survey of the subject and an account of some of the latest experimental work.

The widespread use of the "carbon microphone"—it is employed almost exclusively throughout the world in commercial telephone service—is due primarily to its unique property of being its own amplifier. In converting acoustical into electrical waves, it magnifies the energy about one thousandfold. Other microphones, such as the condenser or electromagnetic type, are unable to do this and so require separate amplifiers when used in practice. For this reason, it seems unlikely that the carbon microphone will be supplanted in the near future for at least the great bulk of telephone work.

The essential element of this device is what has come to be called the "loose contact"—or, as its name implies, a

* Presented at a meeting held Thursday, March 2, 1933.

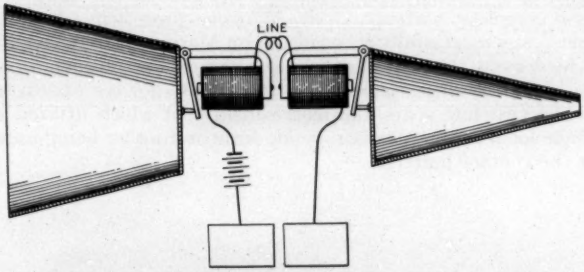
contact between two conductive solids, metals as well as carbons, held together with small forces. The ability of "loose contacts" to transmit speech was discovered independently by Emile Berliner in this country and Professor D. E. Hughes in England. Following Hughes' discovery, Mr. Spottiswood, the president of the British Association in 1878, described it thus: "The microphone affords another instance of the unexpected value of minute variations—in this case, electric currents; and it is remarkable that the gist of the instrument seems to be in obtaining and perfecting that which electricians have hitherto most scrupulously avoided, viz., 'loose contacts.'" Hughes applied the word "microphone" to his instrument because of its remarkable "ability to magnify weak sounds." The word itself is a revival of a term first introduced by Wheatstone in 1827 for a purely acoustical device developed to amplify weak sounds. Although originally confined to the "loose contact" type of instrument, the term microphone has more recently been used—particularly in broadcast, public address, and sound picture work—for any device which converts sound into corresponding electric currents.

EVOLUTION OF THE CARBON MICROPHONE.

The story of the development of the "loose contact" type of microphone is a fascinating one and, although it is beyond the scope of this paper,[1] I should like to refer briefly to a few of the stages in the evolution of the present day instrument. You will recall that Bell's original telephone (Fig. 1) was electromagnetic in principle and acted both as a transmitter and as a receiver. It was, however, very inefficient and Bell himself suggested that some other principle such as that of variation of electrical resistance might overcome the difficulty. He therefore devised the liquid transmitter in which a small platinum wire (Fig. 2), attached to a drumhead of gold-beaters skin, is dipped into a small quantity of acidulated water in a conducting cup. The extent of the area of contact between the liquid and the wire is altered by the motion of the latter, thus altering the resistance in a

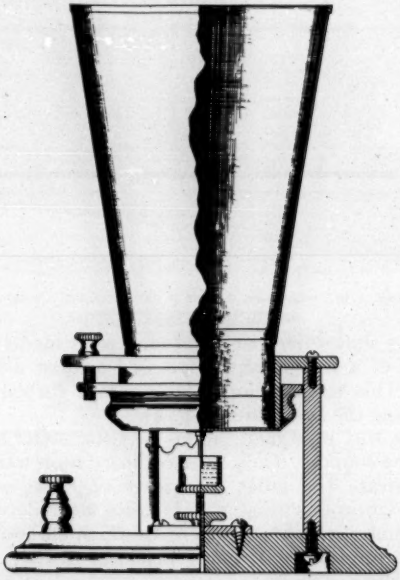
[1] For a more complete account see paper by Mr. H. A. Frederick, "The Development of the Microphone," *Bell Telephone Quarterly*, July, 1931.

FIG. 1.



Bell's conception of the telephone resulted in this sketch which was used in his first patent application of 1876.

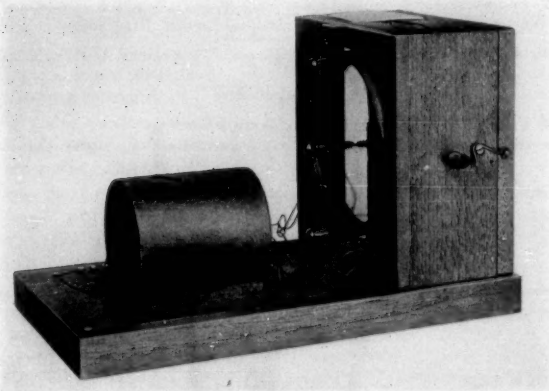
FIG. 2.



Bell's liquid transmitter. With this instrument Bell transmitted the historic sentence, "Mr. Watson come here, I want you."

continuous manner. It was with this instrument that the first complete sentence, "Mr. Watson come here—I want you," was successfully transmitted on March 10, 1876. This achievement stimulated others to work on the problem of a variable resistance element and many new devices appeared in the next few years, the most sensitive of which utilized a single loose contact, carbon in one form or another being used as the contact material.

FIG. 3.



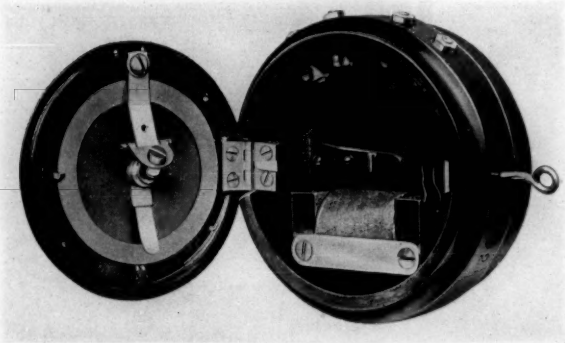
Berliner's first single contact microphone, invented in 1877, employed a metal-to-metal contact.

Figure 3 shows Berliner's first successful model consisting essentially of a metal contact pressed against a metal diaphragm. This was developed later into a carbon-to-carbon contact along the same lines (Fig. 4).

Hughes, too, used metal in his first successful attempt at transmitting sounds. Only three ordinary nails were required to demonstrate the great sensitivity of loose contacts to acoustical vibrations (Fig. 5). Hughes later developed the pencil type of microphone (Fig. 6) in which carbon was used. It was the forerunner of many practical devices developed along this line.

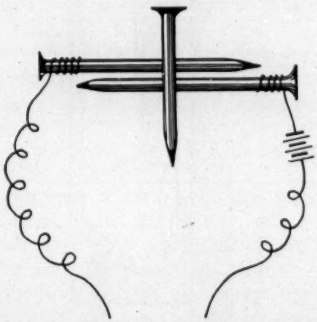
More rugged, reliable and permanent than either of these types was the Blake transmitter shown in Fig. 7. It utilized a metal-to-carbon contact and it owed its success to the mechanical control of the contact pressure. This instrument was used for many years by the Bell System.

FIG. 4.



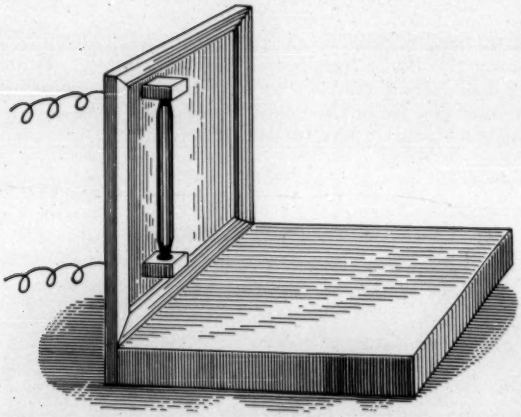
This carbon-to-carbon single contact transmitter was brought out in 1879 by Berliner.

FIG. 5.



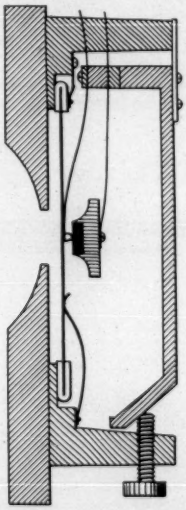
In 1878 Professor Hughes demonstrated the microphonic properties of nail contacts.

FIG. 6.



Carbon pencil type microphone, mounted on a sounding board, was demonstrated by Hughes in 1878.

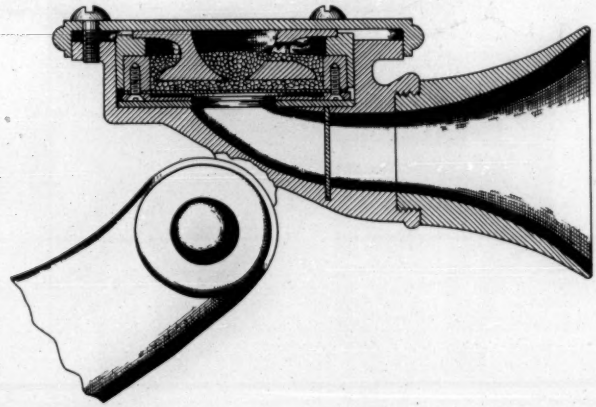
FIG. 7.



The Blake transmitter used a platinum contact pressed against a carbon block.

Then came the Hunnings or the first of the granular carbon microphones (Fig. 8), the immediate ancestor of the granular carbon type used today. Hunnings used powdered "engine coke." It carried more current than the Blake transmitter but it was liable to "pack" and become insensitive.

FIG. 8.

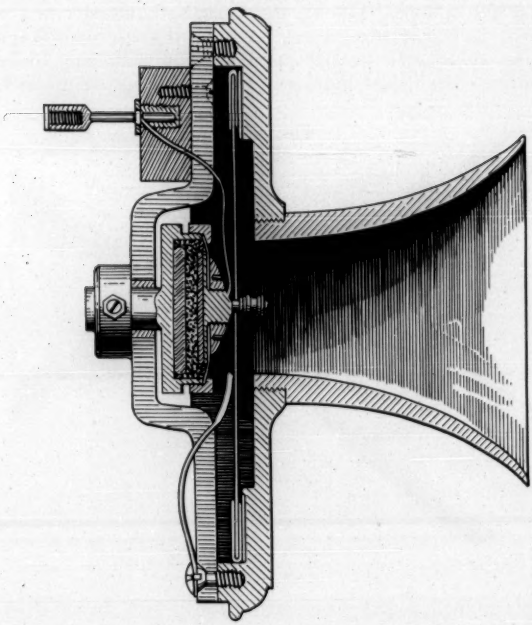


Commercial model of the early Hunnings transmitter in which granular material was first used.

This difficulty was overcome in the design invented by White in 1890, called the solid back type (Fig. 9). Millions of these are used today in the ordinary desk-stand instrument. In this, carbon granules are compressed between two polished carbon electrodes which are immersed in the granular mass in such a way that the particles have more freedom of movement than in the Hunnings instrument. This relieves excess pressure without undue packing.

In Fig. 10 we have a cross-sectional view of a modern handset transmitter. This instrument, which is designed to operate in a wide variety of positions, follows the Hunnings' type in that the granular mass rests against the diaphragm but it differs from it in that the diaphragm does not act as an electrode. Both electrodes, separated by an insulating bar-

FIG. 9.



The solid back transmitter was invented by White in 1890.

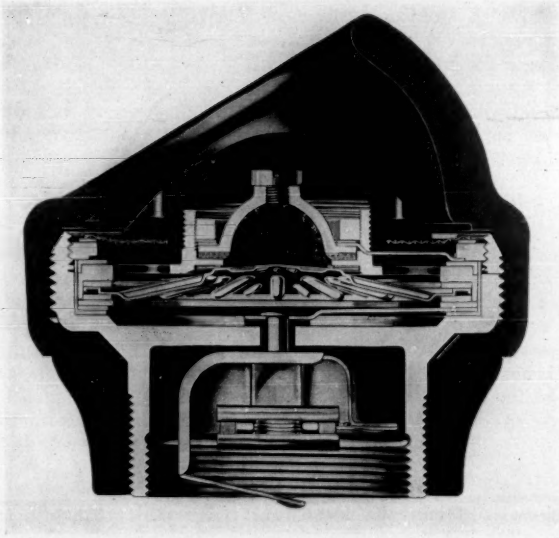
rier, form part of the containing walls of the cell holding the carbon. This is the type which has recently been studied in detail and of which a two dimensional model is shown in Fig. 26.

The carbon used in these instruments is made by a heat treatment of anthracite coal. The particles are about 0.01 inch in size and when magnified they look just like lumps of coal taken from the domestic pile (Fig. 11).

SPECULATIONS OF THE EARLY INVENTORS.

Part of the difficulty in elucidating the microphonic action of the "loose contact" arises because so many effects

FIG. 10.

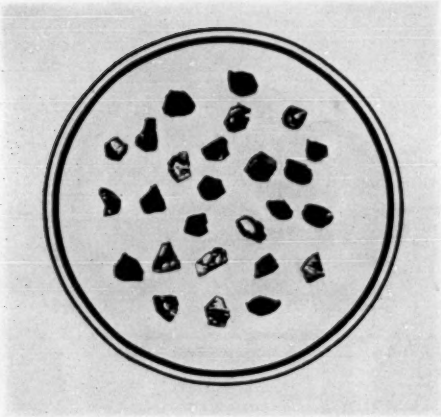


Cross-section of the barrier type transmitter used in modern handset instruments.

can be observed or are associated with the action that it is hard to determine which of them is essential. It is therefore not surprising that there was great diversity of opinion amongst the early inventors.

For instance, experiment shows that contacts tend to move apart when in the act of transmitting sound. This led many, amongst them Berliner, to hold the view that an air film is necessary for microphonic action, that the current somehow passes through the film, and that the variation of the current is due to the variation of the thickness of the film. This view, however, was partly discredited by experiments showing that the moving apart was probably due to a heating of the contact through the passage of current and hence that it is not a necessary accompaniment of microphonic action.

FIG. 11.



Carbon granules made from anthracite coal (× 15).

Again, when one listens through a receiver placed in a circuit containing a "loose contact," noises are heard, especially when the voltage across the contact or microphone is large. These noises are irregular like frying or crackling. Also, if a contact be viewed under a microscope, bright spots are sometimes seen. These facts have led many to think that small arcs are always present and are responsible for microphonic action. Hughes was very much inclined to this view.

There were reasons for supposing that the heating of the contact is a necessary factor in microphonic action. This point of view was supported by Preece, who wrote in 1893, "Indeed there are many phenomena such as hissing and humming that are clearly due to what is known as the Trevelyan effect, that is, the motion set up by expansion and contraction of bodies which are subjected to variation in temperature. This at least tends to favor the heat hypothesis as does also the fact that with continuous use some transmitters become essentially warm."

Another view was that microphonic action arises from change in resistivity of the solid carbon resulting from strain. This view was held by Edison who doubtless believed it because of the success of his microphone which was designed with the object of applying pressure variation to a solid carbon block. It failed of general acceptance because the effect of pressure on resistance, as shown by experiment. seemed definitely to be too small. It was generally considered that the Edison instrument was in fact a "loose contact" although Edison himself did not realize it.

Others of the early inventors considered the contact area to be the essential element—that is to say, the extent of surface or the number of molecules involved in intimate contact. As Professor Sylvanus Thompson expressed it in 1883, "An extremely minute motion of approach or recession may suffice to alter very greatly the number of molecules in contact. . . . Just as in a system of electric lamps in parallel arc the resistance of the system increases when the number of lamps is diminished and diminishes when the number of lamps connecting the parallel mains is increased, so it is with the molecules at the two surfaces of contact."

RECENT THEORIES.

The first attempt at a quantitative theory of microphonic action was made by Professor P. O. Pedersen in 1916.[2] He assumed that microphonic action is due to the variation of the contact area arising from the elastic deformation of the contact material by pressure. Considering the case of two elastic conducting spheres brought into contact, Pedersen assumed that the resistance is made up of two parts; viz., (1) the resistance of a conducting film having a specific resistivity differing from bulk carbon and independent of pressure, and (2) the so-called "spreading resistance" or that which is caused by the concentration of the current flow within the region of the contact area and which would exist independently of any film.

This theory results in a quantitative expression[3] for the dependence of the contact resistance on the force holding the

[2] *The Electrician*, Jan. 28–Feb. 4, 1916.
[3] $R = AF^{-2/3} + BF^{-1/3}$.

contacts together. Pedersen tested it by experiments on carbon spheres and found reasonable agreement over a wide range of force. However a very similar expression can be obtained without postulating the existence of the high resistance film. We have merely to suppose that contact does not take place over the whole contact area owing to surface roughness (the existence of which can be observed under a microscope, especially in the case of carbon).

Mr. F. Gray of Bell Telephone Laboratories worked out an expression [4] based on this assumption which was so nearly like Pedersen's that it was difficult to discriminate between them experimentally. He assumed both that the number of microscopic hills in electrical contact increases as the contact force is increased and that the resistance per hill varies in accordance with the theory of spreading resistance as assumed by Pedersen. His equation was found to fit experimental curves remarkably well for contact forces which are relatively larger than those holding the granules together in a microphone. In the range of smaller forces, however, marked departures from theory were found, the measured value of resistance decreasing too rapidly with an increase of force. Although these departures were believed to be due at least in part to a plastic deformation of the contact material, it appeared possible that other factors come into play and may even be dominant in this region of small contact forces.

For instance, it had been demonstrated that adsorbed films of air are capable of producing a marked increase in the resistance of granular carbon contacts. This revived the air film theory as a possibility under the condition of small contact forces.

Again there is a marked decrease in the resistance of granular carbon contacts with increase in voltage which had not been satisfactorily explained. This fact suggested amongst other possibilities that the conduction process may involve the passage of electrons across gaps of molecular dimensions in the manner of a cold point discharge. Field gradients of sufficient magnitude to extract electrons from a solid must exist in these gaps with only a fraction of a volt across the contacts. If this is the main process by which

[4] $R = AF^{-7/9} + BF^{-1/3}$ (*Phys. Rev.*, **36**, 375, 1930).

current passes between contacts, microphonic action might well be associated with a variation of the gap dimensions under strain.

Again recent work on the theoretical strength of solids had led to experimental results showing that under certain conditions solids may, without fracture, be subjected to strains greatly exceeding those heretofore obtained. This suggested the possibility that the microphonic effect of contacts might after all be associated with the straining of small junctions welded under pressure and current.

In view of the speculative nature of the situation it was clear that a new experimental attack on the problem was necessary. We have been making such an attack during the last few years and I now turn attention to some of the experimental results and the main conclusions to be drawn from them.

RECENT EXPERIMENTAL WORK.

Statement of the Problem.

Since the essential element in the carbon microphone is the so-called "loose contact" the first and most fundamental step toward the understanding of the physics of microphones is the solution of the problem of the "loose contact" when in its sensitive or microphonic state.

Measurements on microphones such as the handset have enabled us to specify pretty accurately the conditions under which any two granules within the structure operate when the microphone is transmitting speech or sound.

In addition to the voltage, which is limited to one volt per contact, these conditions may be stated briefly either in terms of contact forces or in terms of movements between centres of granules. When you realize how small these are—particularly the movements between centres of granules—you will, I think, not be surprised that the solution of the problem of the "loose contact" has been so long delayed.

For the condition of reasonably loud speech the diaphragm motion is about

$$1 \times 10^{-5} \text{ cm.},$$

which is just on the limit of resolution of the highest-power microscopes. It follows from a consideration of the number

of granules in series that the movement between centres of granules would not be greater than 1/10th of this, viz.,

$$1 \times 10^{-6} \text{ cm.},$$

which is in the submicroscopic range. We must, therefore, be able to control and measure movements at least as small as 10^{-7} cm.; not an easy thing to do with a "loose contact."

The contact forces are on the average somewhat less than 10 dynes when the aggregate is in the unagitated state. In the presence of acoustic waves, variable forces of several dynes are superimposed on these fixed forces. The variable forces are smaller than the fixed forces, so that the granules will on the average remain in contact throughout any reversible cycle. We have reason to believe that 10 dynes is about the maximum force which is attained at any one contact during a stress cycle. We must therefore be able to control contact forces within the range 1 to 10 dynes.

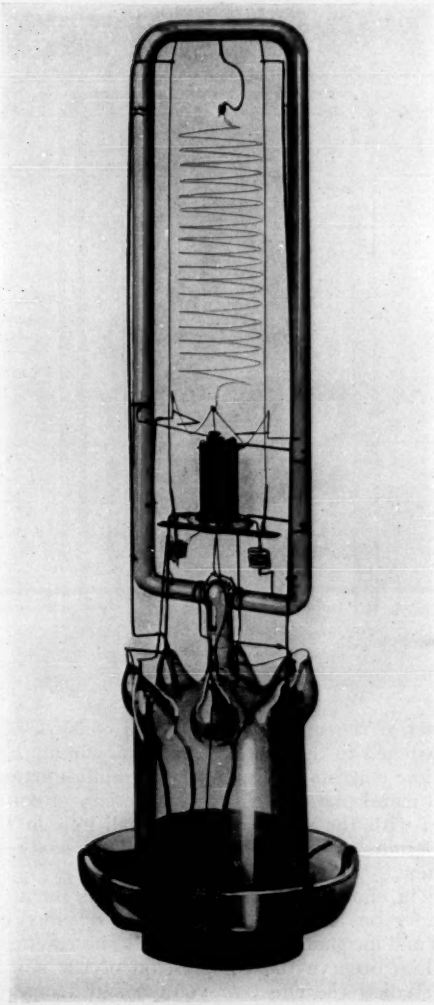
Apparatus and technique have now been developed for studying single contacts within the prescribed range of forces and displacements, and significant measurements have been made which I will now endeavor to describe to you somewhat in detail.

Single Contact Studies.

Figure 12 shows the construction of one of the contact tubes used in this study.

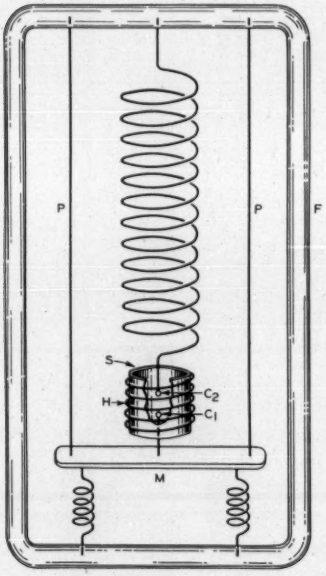
Its essential features are shown diagrammatically in Fig. 13. The contact pieces C_1 and C_2 are fastened respectively to a movable base M and to the lower end of a helical spring made of fused quartz. The base is supported from a fixed frame by two vertical platinum wires P and two stretched springs as shown. The lower contact piece is thus moved by heating or cooling the platinum wires through the passage of current. In this way the contacts may be made or broken and any desired contact force applied, the measure of the force being the compression of the helical spring. The temperature of the contact is varied by surrounding the contact region with a metal cylinder S which may be heated by means of radiation from a coil of platinum wire H, the temperature within the cylinder being measured by means of a thermocouple placed near the contacts.

FIG. 12.



Device for controlling force and temperature used in the study of single contacts.

FIG. 13.

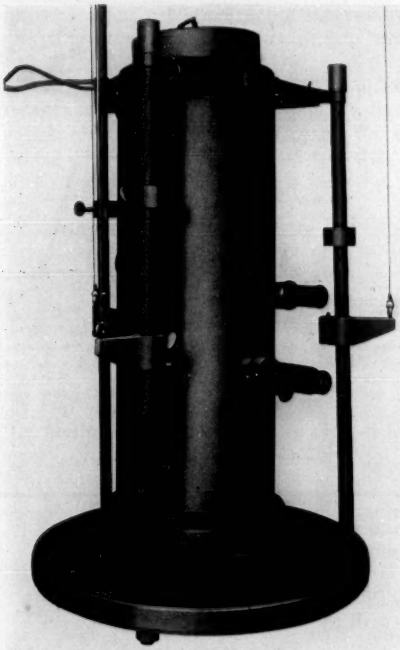


Diagrammatic view of single contact device shown in Fig. 12.

In practice the upper contact piece consists of a single granule fastened to the end of a platinum wire and the lower contact piece consists of a number of granules attached to a horizontal metal plate; in this way a variety of contacts can be studied with the same tube. A small hole in the metal cylinder permits of direct observation of the contacts during measurement.

Figure 14 shows how the apparatus was mounted in an iron cylinder on a damped suspension to protect it from acoustical and mechanical disturbance. The two microscopes were used to observe the compression of the silica spring. We first studied the effect of voltage and temperature on

FIG. 14.

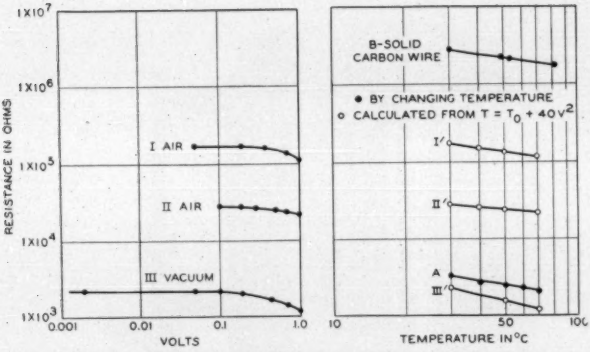


The single contact device is mounted in a heavy container on a spring suspension to minimize acoustic and mechanical disturbance.

contacts held together with constant forces. Reversible characteristics could in all cases be obtained for voltages up to 1 volt and for temperatures up to about 80° C.

Typical characteristics are shown on Fig. 15 in which the contact forces were of the order of 1 dyne. On the left are plotted the resistance-voltage characteristics and on the right the resistance-temperature characteristics. All of the variables are plotted for convenience on logarithmic scales.

FIG. 15.



Characteristics showing the effect of voltage and temperature on contact resistance.

The curves *I*, *II* and *III* illustrate the fact that Ohm's law is found to hold for all contacts up to about 0.1 volt and that above these values the contact resistance decreases with increase of voltage. The fractional decrease in resistance with voltage above 0.1 volt is independent of the contact resistance and whether or not the measurements are made in air or vacuum.

In curves *I′*, *II′* and *III′* we have changed the voltage scale of the curves *I*, *II* and *III* to a temperature scale in accordance with the relation,

$$T = T_0 + 40V^2.$$

This relation has a theoretical basis [5] in the Joule heating of the contacts due to the passage of current and contains the assumption of a value of Wiedemann Franz ratios characteristic of solid carbon.

[5] This theory, based on earlier work of Kohlrausch, was worked out in useful form independently in Bell Telephone Laboratories (unpublished work) and by R. Holm (*Zeit. Tech. Phys.*, 3, 1922). It gives the approximate relation const. $\frac{V^2}{K_0/\sigma_0}$, as the increase in temperature above room temperature, where V is the contact voltage, and K_0/σ_0 the Wiedemann Franz ratio for the contact material.

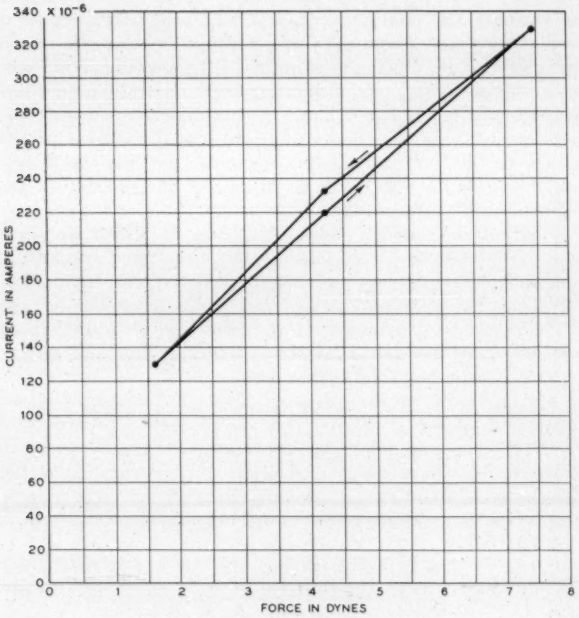
These curves have substantially the same slope as A, which is a characteristic measured by heating a contact in the furnace, the contact voltage being sufficiently small to avoid appreciable heating of the contact due to this cause, and also with B, which was obtained with a solid carbon wire produced in a manner to simulate closely microphone carbon. We are able to conclude from measurements such as these that the nature of the conducting portions of contacts is that of solid carbon both for air and vacuum and that the departures from Ohm's law—at least up to 1 volt—are due to the Joule heating of the contacts.

From measurements similar to these in which we show that the admission of air has no effect on the temperature coefficient of resistance—although it produces a marked increase in the resistance at any particular temperature—we are also able to conclude that the presence of adsorbed air does not alter the nature of the conducting portions of the contacts but merely limits their areas.

Turning now to the effect of contact force on contact resistance: we see (Fig. 16) that large and approximately reversible resistance changes are produced as the force is varied repeatedly between fixed limits. This shows that the effect is in the main elastic, though the existence of a narrow loop indicates a small plastic or irreversible movement as a secondary effect.

We have thus established that the current is conducted through solid carbon and that the deformations are mainly elastic. These facts give strong support to the "elastic theory" of "loose contacts," i.e., the hypothesis that the change of resistance takes place because of a change in contact area under pressure. An extensive study of the resistance-force characteristics gave results which could not be simply interpreted (just as Gray had found) and, because of the possibility that unknown cohesional or frictional forces were involved, the work was extended by a study of resistance-displacement characteristics. Through a comparison of the two sets of data we were led to the conclusion that the stress-strain characteristics are not so simple as those assumed in Pedersen's or Gray's analysis, indicating a study of the elastic behavior of contacts as the most promising line of attack on the problem.

FIG. 16.



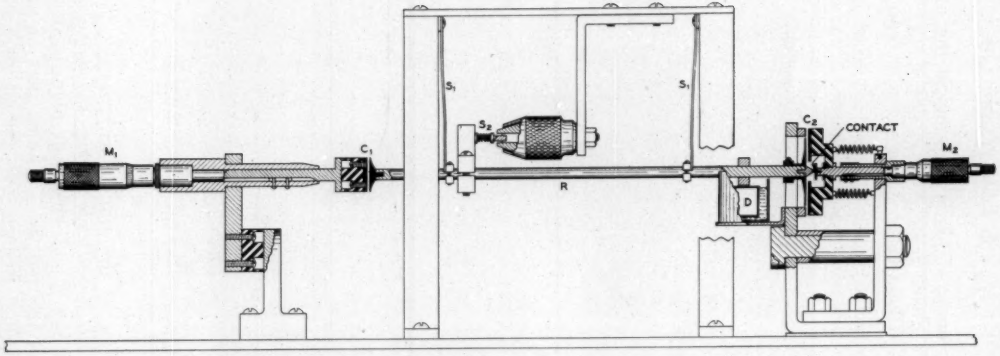
Typical current-force cycle obtained with a single contact.

Figure 17 shows the mechanical system developed for this purpose. With it known forces can be applied to a contact element and at the same time its movement can be measured.

The contact is made between a carbon granule and a polished carbon plate, the granule being attached to the end of a rod R suspended by springs S from a fixed frame and the plate being attached to the end of a micrometer screw M_2 capable of giving to it a translational motion without rotation.

The force is applied to the granule electrostatically by means of voltage applied between the condenser plates C_2, one of which is attached to the rod R and the other to the

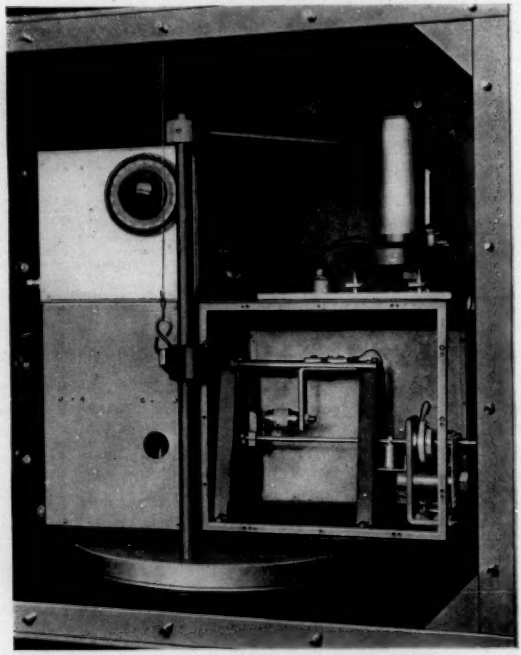
FIG. 17.



Diagrammatic view of mechanical system used in the study of single contacts.

micrometer screw. This is in principle the attracted disc electrometer of Kelvin and it is capable of applying forces up to 15 dynes without using voltages greater than 200.

FIG. 18.

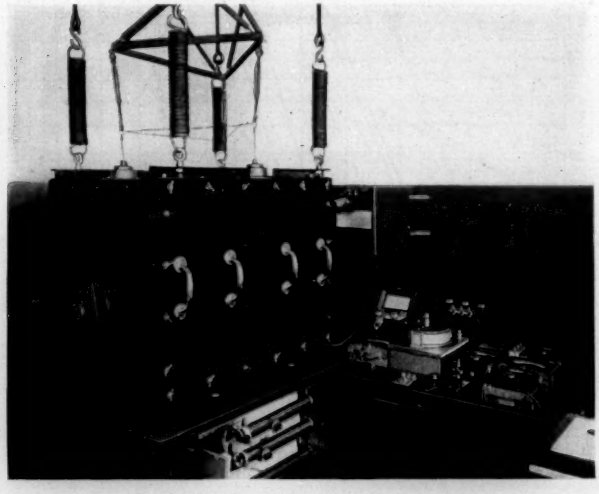


Mechanical system and associated electrical apparatus as set up for single contact study.

The motion of the granule with respect to the carbon plate is measured electrically through the variation of capacity of the condenser C_1, of which one plate is attached to the other end of the rod R. C_1 forms part of an oscillating circuit of natural frequency n_0 (about 2000 kc.) which is

coupled to a wave-meter circuit adjusted for oscillation at a frequency n_1 slightly different from n_0. Changes in the frequency arising from the changes in capacity C_1 alter the energy picked up by the wave-meter circuit and this energy, which is recorded by means of a galvanometer, serves as a measure of the change of capacity or motion of the rod R. With this arrangement it is possible to measure motions as

FIG. 19.



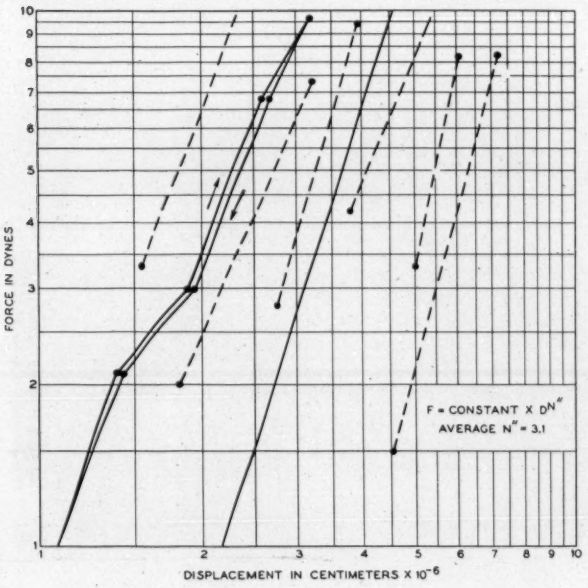
Exterior view of complete experimental arrangement.

small as 1×10^{-7} cm. and under the best conditions as small as 1×10^{-8} cm. It is necessary to have good damping, which is obtained by means of immersing the drum D in polymerized castor oil. The accessory spring S_2 is used merely for calibrating purposes.

Figure 18 shows the appearance of the apparatus as set up for measurement. The condenser is contained in the lower housing at the left, the wave-meter in the upper housing.

The whole apparatus including the galvanometer is supported on a delicate spring suspension within a second large lead container, the frame of which just appears at the edge of the photograph and which is also supported by springs.

FIG. 20.

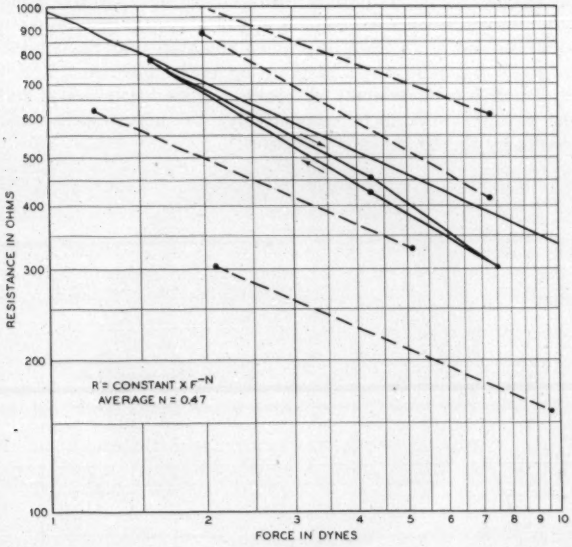


Typical force-displacement characteristics of carbon granules pressed against a polished carbon plate.

Figure 19 shows the appearance of the complete setup with the cover on the outside container. This begins to compete with cosmic ray apparatus from the point of view of the amount of lead involved, the outer container weighing about 600 lbs. Port-holes—one of which appears on the near end of the box—permit adjustments to be made on the

apparatus within, thus eliminating the necessity for removing the large outer cover which, as you may surmise from the number of handles, requires the combined efforts of two men to remove it. All of this protection is, of course, to shield

FIG. 21.



Typical resistance-force characteristics of carbon granules pressed against a polished carbon plate.

the apparatus from mechanical vibrations and acoustic disturbances.

With this apparatus we investigated the variations in displacement and resistance when the forces are varied cyclically between fixed limits. Measurements on a large number of contacts are summarized in curves, Figs. 20 and 21. The cyclic characteristics, though somewhat irregular and having the form of narrow loops, approximate straight lines when the variables are plotted on logarithmic scales. Only

one complete characteristic is shown in each set of curves, other typical measurements being represented by dotted straight lines joining the end points of their respective cycles. The full line in each figure represents the cycle of a typical contact, obtained by averaging, over the range in which the difference between the maximum and minimum force limits or maximum and minimum displacement limits is relatively large, in which case the slope is apparently constant.

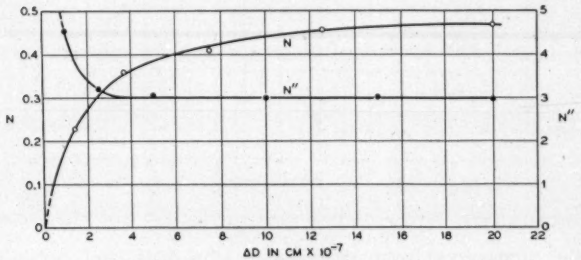
If we let N'' and N represent the slopes of the typical force-displacement and resistance-force characteristics and if F, D and R be the contact force, the contact displacement and the contact resistance, respectively, we may express our results by the approximate relations:

$$F = \text{const.} \cdot D^{N''}, \tag{1}$$

$$R = \text{const.} \cdot F^{-N}. \tag{2}$$

The values N'' and N are not, however, independent of the force or displacement limits when these limits are relatively small. In Fig. 22 we have plotted values of N'' and N as

FIG. 22.



Effect of the extent of contact motion (ΔD) on N and N''. (Average values.)

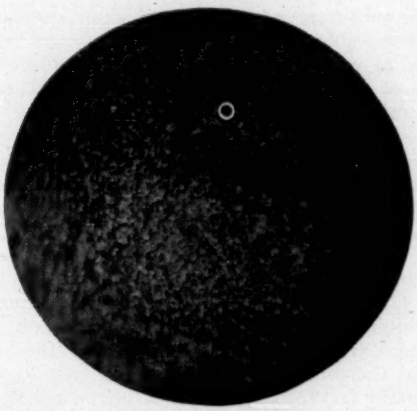
functions of the difference between the maximum and minimum displacement (ΔD). We see that for relatively large values of ΔD, N'' and N approach the limiting values 3.1 and 0.47, respectively, but for smaller values of ΔD, N'' becomes greater than and N less than its limiting value. The limiting value of N'' is greater than that which would be

obtained through the contact of hemispherical surfaces and represents a more rapid stiffening of the contact with compression.

We will first give our attention to the limiting value of N''.

A consideration of the nature of contact surfaces as revealed by the microscope furnished the clue to the interpretation of our results. A typical surface is shown in the photomicrograph (Fig. 23). Evidently it is very hilly, the hills being much the same size and height. The magnification ($\times$ 2400) is such that the small white circle has a diameter of 8×10^{-5} cm. and it is clear that the circle encloses several hills.

FIG. 23.



Photomicrograph of the surface of a carbon granule ($\times$ 2400).

From the theory of elasticity we may deduce that if two hemispherical hills of carbon having a radius of the order 1×10^{-5} cm. are brought together with forces of the order of 1 dyne the maximum stresses will probably not exceed the elastic limit of carbon and hence that the hills will deform elastically. The motion involved in such a deformation will be of the order of 1×10^{-6} cm. and if other hills are encountered, as is most probable with such a movement, the

stresses will be shared and hence the stress per hill reduced. According to this view forces larger than one dyne can be applied without exceeding the elastic limit merely by virtue of the distribution of the hills which will come in to share the stresses. Furthermore, such a contact will stiffen up more rapidly with compressional displacement than will a contact made on a single hill. This concept of a loose contact, therefore, seemed to offer possibilities in the way of an adequate explanation of the experimental results.

At first the problem seemed too complex for mathematical analysis and a study of the elastic behavior of contact surfaces having various arrangements of little hemispherical hills was made with the aid of large scale rubber models. Quarter inch rubber balls were cut in half for this purpose and arranged on bases of suitable material and shape.

In Fig. 24 we have plotted the force-displacement characteristics of three different surfaces: *I*, that of a single smooth hemisphere; *II*, that of small hemispheres of equal height evenly distributed on a portion of a large 32 inch sphere made also with rubber; and *III*, that of hemispherical surfaces of random height fastened to a flat plate, about 100 hemispheres being used.

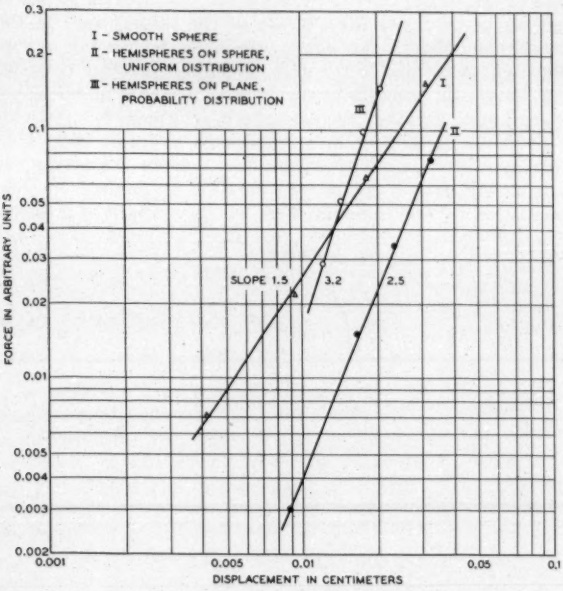
We see from the slopes of these curves that the model made with hills of random height on a flat plate behaves most like the actual contacts, the slopes of the corresponding curves being 3.2 and 3.1, respectively. This arrangement is also the one which most nearly represents the carbon surfaces as viewed under the microscope. Here the hills have various heights and the radius of the underlying base (0.015 cm.) is so much larger (1000 fold) than that of the average hill that within the region of the contact area the surface of the former may be regarded as plane.

The slope of curve *I* is in accord with a formula derived from the theory of elasticity by Hertz connecting the force F pressing together two elastic spheres and the movement D between the centres of the spheres:

$$F = \text{const. } D^{3/2}. \qquad (3)$$

The constant includes such factors as the elastic moduli of the contact materials and the radii of the spheres and need

FIG. 24.



Stress-strain characteristics obtained with contact surfaces made of rubber.

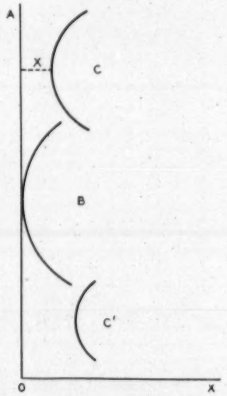
not concern us here. The case of a sphere pressed against a flat plate, as in our experiments, is a particular case of this general equation, the constant only being affected.[6]

The slopes of curves *II* and *III* are also in accord with theory, as we shall see, when one makes the simple assumption that the elastic deformation is confined to such a small region near the contact in each hill that the underlying base is not

[6] Formula (3) is known to hold accurately for values of D not greater than about 1 per cent. of the radius of the sphere (J. P. Andrews, *Phys. Soc. Proc.*, Vol. 42, No. 236). This condition is fulfilled in the case of curve *I* but D is as great as 10 per cent. of the radius in the case of a few of the hills involved in the maximum compression shown in curves *II* and *III* (Fig. 24).

appreciably deformed. This assumption was tested in the case of the model having the spherical distribution of hemispheres by changing the stiffness of the rubber used in the underlying sphere. No effect was produced on the stress-strain characteristic (curve *II*). We may therefore consider that the elastic reactions produced in each hill are independent of each other and that the base is not deformed, so that with a given distribution of hills it becomes a simple matter to calculate their combined effect over a given compressional range. We may represent the conditions essential for our calculation by the diagram, Fig. 25, in which *A* represents

FIG. 25.



Schematic representation of a rough surface used in mathematical analysis.

the plane surface of the smooth contact element just making contact with the highest hill of the rough contact element. Under compression, *A* may be considered as moving in the direction of its normal *x*, compressing *B* and, with increasing motion, coming into contact with other hills *C* and compressing them according to relation (3). The position of *C* is conveniently defined by its distance *X* from the plane *A*.

Any continuous distribution of hill positions, typified by *C*, which would be encountered through a small compressional

movement, may be approximately represented by the expression,

$$N_x = \text{const. } x^n, \tag{4}$$

where N_x is defined as the number which multiplied by dx gives the number of hills coming into contact with the plane when it moves from x to $x + dx$. The exponent n is a constant which for convenience we may call the distribution constant.

For a total compression D the $N_x dx$ hills will be compressed an amount $D - x$, and hence the total force of reaction F is given by

$$F = \text{const.} \int_0^D x^n (D - x)^{3/2} dx,$$

which integrates to the form,

$$F = \text{const. } D^{n+5/2} = \text{const. } D^{N''}. \tag{5}$$

The constant here includes a summation of the individual constants of equation (3). It is clear that if the hills have different radii the constant only will be affected, so that equation (5) may be regarded as general in this respect.

For the case of uniform hills distributed on the surface of a sphere it may be shown that equal numbers of hills will be added for equal increments in x; in which case $N_x = \text{const.}$[7] From this it follows that $n = 0$ and $N'' = 2.5$ in agreement with the measured value, curve *II*.[8] For $N'' = 3.2$ as obtained with the hemispheres of random height on a plane, curve *III*, n would have the value 0.7. The corresponding distribution function N_x would approximate to that of the portion of an ordinary error curve near its maximum. A rough determination of the distribution of heights amongst the small rubber hemispheres showed in fact that they approximated closely to an error curve and that the displacement range covered that portion of the curve near the most probable height.

[7] This argument rests on the fact of geometry that if A is the area of contact between a sphere of radius r and a plane, $dA/dx = 2\pi r$.

[8] This agreement between theory and experiment shows that the compression of some of the hills by an amount in excess of 1 per cent. of their radii has not affected the applicability of equation (3) to our problem.

It would appear from this analysis that the elastic behavior of our carbon contacts under conditions of relatively large strain is adequately explained on the very simple assumption that the hills which we observe under the microscope have a random distribution of heights and behave like smooth spherical surfaces. We have, however, still to account for the hysteresis and the large values of N'' corresponding to small values of ΔD as well as the values of N (Fig. 22).

It is unlikely that the hills which we observe under the microscope are submicroscopically smooth, in which case we would expect a small plastic movement in these secondary hills arising from overstrain. We have direct evidence for this in the fact that contacts once established—even without the passage of current—require relatively small but finite forces to break them. Such junctions within the contact region could well account for hysteresis and a stiffening up of the contact in the region of small strains. Furthermore it is to be expected that they might affect the resistance behavior to a much greater extent than the elastic, and over a wider range of strain, since the junctions—though too weak to affect appreciably the contact stiffness—might well carry a relatively large proportion of current; in which case the value of N would be smaller than that calculated on the assumption of smooth spherical surfaces.

We will now derive an expression relating resistance and force for the type of contact considered in the derivation of equation (5), assuming smooth hills.

Classical theory [9] gives the following formula for the conductance $1/r$ of the contact formed by compressing, by an amount D, a single smooth conducting sphere against a flat conducting plate,

$$\frac{1}{r} = \text{const. } D^{1/2}. \tag{6}$$

It appears reasonable to assume that the hills which come into contact with compression act independently of each other as regards conduction. The conductances may there-

[9] Riemann Weber.

It is here assumed that the mechanical and electrical areas of contact are coincident, which according to the ideas of wave mechanics may not be the case.

fore be added and we may write for the total conductance $(1/R)$ produced by a compression D involving many hills:

$$\frac{1}{R} = \text{const.} \int_0^D x^n (D - x)^{1/2} dx,$$

which integrates to the form,

$$\frac{1}{R} = \text{const.}\ D^{n+3/2},$$

which in combination with (5) gives

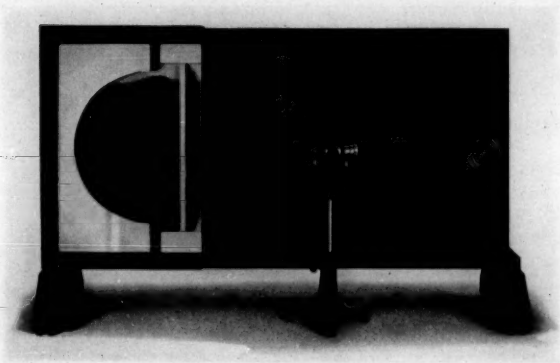
$$R = \text{const.}\ F^{\frac{2n+3}{2n+5}} = \text{const.}\ F^{-N}. \qquad (7)$$

Using the value of n consistent with equation (5) through the measured value of N'', viz., $n = 0.6$, we get $N = 0.67$. The measured value of N (0.47) is, as we have surmised, too small though it is of the right order of magnitude.

We are, of course, investigating the factors which give rise to this discrepancy as they will play an important part in any complete theory of microphonic action, and we are extending our study to the behavior of granular aggregates in simple cells and microphone structures. We have shown that the value of N in a simple cell composed of parallel electrodes is quite consistent with our simple theory for single contacts, and therefore indicates that the behavior of an aggregate of contacts is determined by the behavior of the individual contact. Furthermore, we have shown, through static measurements on the handset instrument, that the granular aggregate within this irregularly shaped structure behaves like the aggregate in a simple cell. We are therefore confident that the behavior of the group will be explained in terms of the behavior of the single contact.

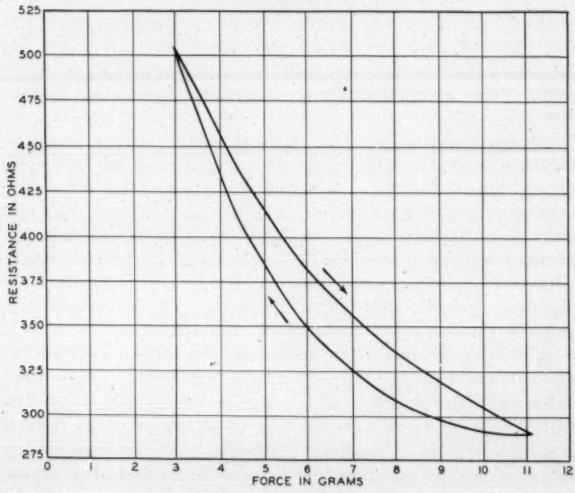
The behavior of the two dimensional model of the handset microphone (Fig. 26) is most convincing in this connection. Although this model was set up originally to study the distribution of stresses in this type of structure it has proved most useful in other phases of our work. Quarter inch rubber balls represent the granular particles of the actual microphone

FIG. 26.



Model of handset transmitter cell.

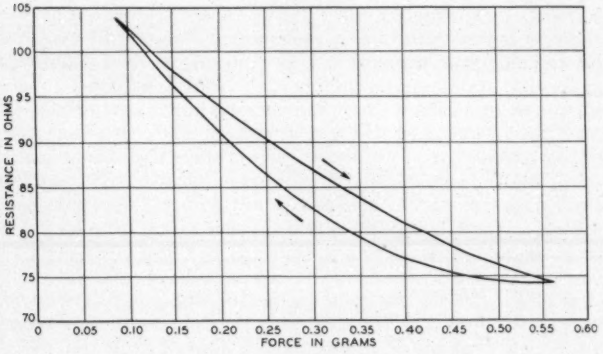
FIG. 27.



Force-resistance cycle obtained with transmitter model.

and by coating these with a conducting layer of graphite and lacquer we are able to make them behave electrically as well as elastically in accordance with our simple theory. When placed in the model the aggregate is compressed cyclically by means of the piston which acts as a diaphragm, producing a change of resistance in the current path around the insulating barrier. The curves shown in Figs. 27 and 28 are typical force-resistance cycles, obtained with the model and the actual instrument under conditions wherein the reactive forces are mainly elastic. The similarity of these character-

FIG. 28.



Force-resistance cycle obtained with a standard transmitter.

istics is striking. The existence of the loops indicates that the reactive forces are not entirely elastic and that the behavior is modified by friction, as in the case of single contacts.

In conclusion it seems fair to say that our experiments on "loose contacts" under conditions which are equivalent to those under which they operate in actual microphones have given a satisfactory picture of the essential nature of such contacts, and their mode of operation when strained, both from the elastic and the electrical point of view. The

electrical current is carried through regions in intimate contact and changes in resistance under strain are due both to a variation in the number of microscopic hills which form the carbon surface and to area changes at the junctions of these hills arising from their elastic deformation in accordance with the well-known laws of elasticity.

RECENT DEVELOPMENTS IN ARCHITECTURAL ACOUSTICS.*

BY

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It was almost exactly nineteen years ago that Franklin Institute was addressed on the subject of Architectural Acoustics by Professor Wallace Sabine of Harvard. The distinguished speaker was well worthy of the distinguished audience that heard him on that occasion. As a lone worker in what was then a hitherto untilled field of investigation, he came to present a record of individual achievement, which, viewed in the perspective of the intervening years, marks him as a truly great scientist. The problems to which as a very young man he had set himself some twenty years before were as old as the architecture of ancient Greece. His solution of those problems came not through the acquisition of new research tools more powerful than those his predecessors had used. Rather it came as the result of the skillful and persistent application of the scientific method to a problem that had not before been the object of any thoroughgoing scientific attack. The necessary instrumental equipment was all available fifty years before Sabine undertook the problem. So far as experimental equipment was concerned a scientist with Sabine's skill, patience, and insight might have anticipated his results by half a century.

Tonight I cannot report any such record of outstanding individual achievement. Rather as one of a fairly numerous group of workers I am here to tell you briefly of the advance that has been made during the last decade and a half along the trail which Sabine blazed some thirty years ago. New and powerful aids to acoustic research have come to hand during that period. It is quite conservative to say that since 1920 more progress in the development of instruments for the

* Presented at a meeting held Thursday, December 7, 1933.

making of acoustical measurements has been made than in the whole prior history of the study of sound.

A whole list of devices which are commonplace today were quite unavailable for acoustical research thirty years ago. Then the physicist had as sources of sound only organ pipes and tuning forks, neither of which had any considerable degree of variability either in pitch or acoustical power. The discovery near the close of the last century of the thermionic effect, and the possibility of the control of an electron stream in vacuo, gave us the three element vacuum tube, and the vacuum tube amplifier. Another application gave us the vacuum tube oscillator, with which may be generated alternating electrical impulses of frequencies covering the entire range from the few cycles per second that generate the lowest pitched sounds up to the thousands of kilo-cycles used in short radio wave transmission. The combination of oscillator, amplifier and loud speaking telephone affords means of producing sounds covering the whole range of audible frequencies, variable in known ratios of intensity from the barely audible to tones of painful intensity.

Equally great advance has been made in the technique of sound intensity measurements employing these same electronic means of current amplification. The condenser microphone with its stretched metal diaphragm was first described by E. C. Wente of the Bell Laboratories in 1917. In this instrument the alternating e.m.f. generated is strictly proportional to the alternating pressure in the sound wave so that with proper amplification one can make sound intensity comparisons by purely electrical measurements. More recently, microphones of the moving coil type designed to give an almost uniform response independently of frequency have been developed, and also, the so-called ribbon microphone in which the vibration in a magnetic field of a very light conducting strip generates e.m.f.'s which are proportional to the particle velocity in the wave.

Under the spur of commercial demands in the arts of telephony, sound recording, and sound reproduction, great strides have been made in the quantitative study of sound in general, many of which have an important bearing on the problems of its control in buildings. One of the most im-

portant of these is our relatively new knowledge of the performance of the human ear in relation to sound intensities. Considered as a sound detecting device, the ear is a very remarkable instrument. It normally responds to vibrational stimuli, ranging in frequency from about 20 to 20,000 vibrations per second. As a matter of practice most ordinary sounds including those of music and speech lie between the limits of 60 and 10,000 vibrations per second. The intensity range between the threshold of hearing and the intensity of the loudest sounds is truly enormous. Although very loud sounds represent only small amounts of power expressed in ordinary units, yet below this maximum we have a very great range of sound intensities in which our auditory experiences lie. Thus a painfully loud tone of 1000 cycles, two octaves above middle C has a power of around 10,000 micro-watts, while one that is barely audible is of the order of 10^{-9} micro-watts per square cm. cross section of the sound wave, an intensity range of about 10^{13} to 1.

Someone has computed that the sensitivity of the ear is so great that it can detect vibrational amplitudes no greater than 1/30 the diameter of a nitrogen molecule. The pressure variation in the least audible sound is roughly equal to the weight of a hair whose length is equal to one-third its diameter.

It has been estimated that if the ear were only slightly more sensitive that it is, it would be possible for us to hear the noise of thermal agitation. In other words the ear would respond to the Brownian movements of the molecules. The capacity of our ears to register sensation from such infinitesimal stimuli and at the same time to withstand the assults of vibrational energies 10 trillion times as great makes it one of the most marvelous physical instruments of which we have any knowledge.

Although it has this great sensitivity to minute sound intensities, the ear is at the same time relatively insensitive, considered as a difference recording device. Its response to differences of sound intensities is roughly logarithimic, and on the average any given sound intensity must be increased by about 26 per cent. of itself before the ear will record a difference of loudness. That is to say, if the logarithm of the physical intensity is increased by 0.1 (logarithm of 1.26 = 0.1)

the ear will barely perceive a difference in the loudness of the sound. This has led to the use of the so-called decibel scale in designating acoustic intensity levels. Two sound intensity levels differ by 1 bel if the logarithm of the ratio of the corresponding intensities is 1.0, i.e., if one is ten times the other. The *decibel* difference between two intensity levels is 10 times the logarithm of the ratio of the intensities.

The following are the intensity levels of some ordinary sounds expressed in decibels above the threshold of hearing.

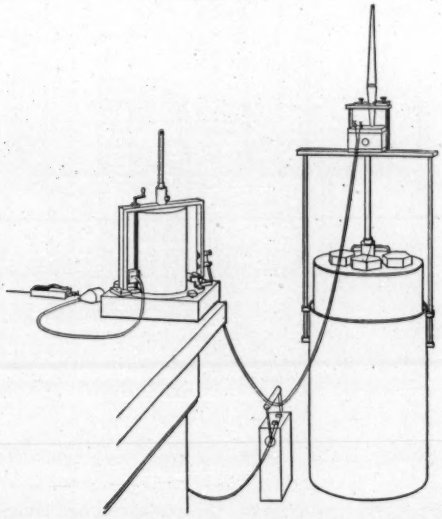
Threshold of hearing		0
Barely audible		0–10
Very faint	Rustle of leaves Whisper	10–20
Faint	Quiet garden Quiet home	20–30
Moderately loud	Quiet office Quiet automobile Vacuum cleaner Ordinary conversation	30–60
Loud	Telephone conversation Loud radio Motor truck Heavy city traffic	60–80
Very loud	Pneumatic drill Elevated train Riveting machine	80–100

The acquisition of this knowledge of the actual amounts of physical energy present in ordinary sounds has helped us enormously in orienting ourselves in quantitative work in architectural acoustics. For example, laboratory tests show that an ordinary wall such as would be used in apartment house construction transmits only about one-tenth of one per cent. of sound between adjoining rooms. An apartment house owner might think such walls would furnish adequate insulation between apartments. But expressing this on the decibel scale, 0.1 per cent. means a reduction in the ratio of 1000 to 1. The logarithm of 1000 is 3, and the decibel reduction is 30. A loud radio or a conversation over the telephone represents a level of about 70 decibels. A thirty decibel reduction lowers this to 40 decibels, a level which is still amply audible and, in the case of speech, quite intelligible.

Fussy tenants will undoubtedly want more insulation than this from the noise of their neighbors.

The developments that have been made in acoustical measurements since Sabine's time may be strikingly shown by a comparison of the means which he employed for the measurement of sound absorption coefficients of materials

FIG. 1.

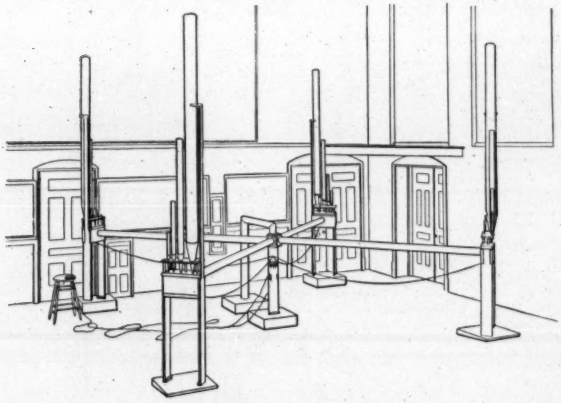


Original apparatus used by W. C. Sabine for measuring the reverberation in rooms. The tank supplies air at constant pressure for operating the single organ pipe. The time during which the sound remains audible is measured on the chronometer.

with those which are available today. The absorption coefficient of a material is the fraction of the sound energy that is *not reflected* when sound is incident upon its surface. Knowledge of this property of all the surfaces that constitute the exposed interior of a room is essential to the successful acoustical design of audience rooms. It should be said that the method employed today is essentially that which Sabine developed and used. The improvement is wholly in the tools

that are employed. In these measurements we must proceed by indirection and measure the sound absorbing efficiency of materials by measuring their effects in reducing the reverberation in rooms. This is known as the Sound Chamber Method. Sound is produced in a large empty room. After the source is stopped the sound persists for an appreciable length of time, its intensity decreasing gradually as the energy is dissipated

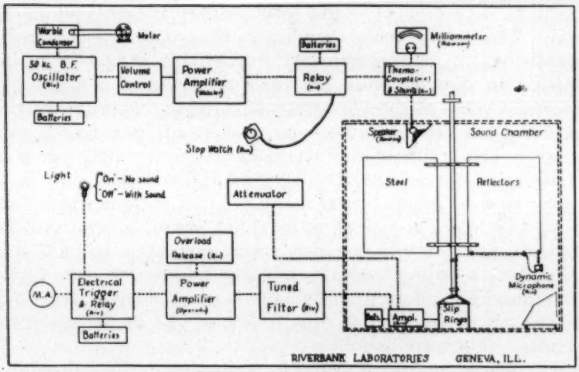
FIG. 2.



Equipment used for securing sounds of known relative initial intensities. Four small organs speaking simultaneously were assumed to produce four times the sound intensity produced by one.

by the absorptive process at the walls. This time, the so-called Reverberation Time, required for the intensity to sink to a given fraction, 1/1,000,000 let us say, of its initial intensity is inversely proportional to the total absorbing power of the bounding surfaces. This total absorbing power is the sum of the products obtained by multiplying the area of each surface by its particular coefficient. From the change produced in the reverberation time of the Sound Chamber by a known area of a given material, the absorption coefficient of the material can be computed, provided we know the absorbing power of the empty room, without the absorbing material present. This can be determined by measuring the times required for

FIG. 3.



Riverbank equipment for measuring rates of decay of reverberant sound in Sound Chamber. Variable pitch and intensity are secured by controlling and measuring the oscillating current supplied to the loud speaker source of sound. The sound is picked up by the microphone. The amplified microphone current operates a light signal by which the timings are made.

two sounds, whose relative initial intensities are known, to decrease to some fixed intensity. The empty Sound Chamber thus becomes our acoustical yard stick. We calibrate it by experimentally determining its total absorbing power or what is proportional to this quantity, the rate at which sound energy dies away in it.

Now, it happens that this method, devised originally for the measurement of the sound absorption coefficients of materials, lends itself most readily to the study of another problem in the control of sound in buildings, namely that of the transmission of sound from room to room by way of intervening wall structures. Calibration of Sound Chamber gives us the rate at which the logarithm of the sound intensity decreases. Now suppose we measure the time which sound remains audible in the sound chamber, and we also measure the duration of audibility of the same sound heard through a test wall built into an opening of this room. The difference between these two times multiplied by the rate of decrease is the logarithm of the factor by which the intensity is diminished by the wall. This logarithm is an excellent measure of the

sound insulating properties of a wall. We can reduce it to decibels reduction in sound level, simply by multiplying by ten. The evergrowing tendency toward congested living conditions in city apartments makes this matter of sound insulation one of great practical importance. During the fourteen years of the life of the Riverbank Laboratories this problem has claimed a very large share of our research program. The result of this work has been to clarify our conceptions of the mechanics of sound transmission, and in so doing to clear away a good many popular notions which turn out to be false, as well as to establish scientific principles for rational procedure in securing sound insulation in buildings. The work has involved a very large number of tests (some four hundred different walls have been studied). Time will permit us to summarize only a few of the more important results. They are as follows:

1. The reduction of sound in transmission through porous flexible materials, such as felts and fiber boards, is an absorptive process, so that the logarithm of the reduction by such materials increases linearly with the increase in thickness.

2. The reduction in transmission of sound by porous materials which are poor absorbers and good reflectors is small in comparison with impervious materials which are good reflectors and poor absorbers. Thus the reduction by 4 inches of hair felt is less than is produced by one inch of solid plaster.

3. The transmission of sound by solid impervious partitions results from the forced vibration of such structures under the alternating pressures of sound waves. These vibrations set up sound waves on the further side of the partition. Sound transmission is therefore a matter of the overall mass, stiffness and frictional damping of the structure as a whole.

4. The sound reduction averaged over the whole range of frequencies produced by simple partitions of masonry (plaster, clay and gypsum tile and brick) is very nearly proportional to the cube of the weight per square foot, regardless of the masonry material of the partition.

5. The sound reduction by double wall construction is conditioned by the degree to which the two units of such constructions are structurally and spatially separated. In case of otherwise complete structural separation, the filling

of the inter-wall space with fibrous or granular fills increases sound transmission. Even slight structural tying increases sound transmission to a marked degree.

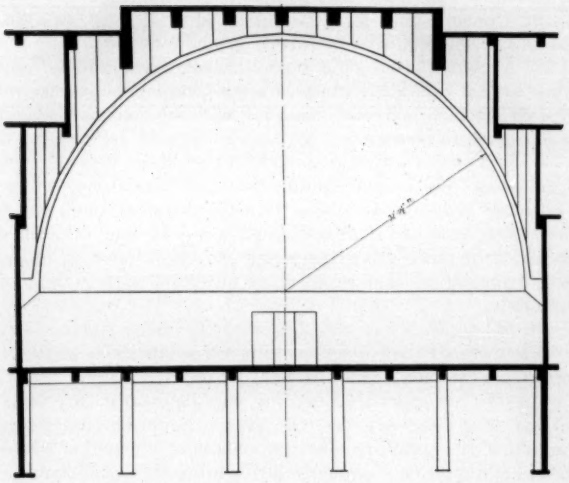
6. A combination of a heavy construction and a light construction elastically coupled is the most effective means of securing high acoustic insulation without excessive weight or excessive thickness.

7. In a broad way, the mass of a partition plays the predominating role in determining its sound insulating value. The sound reduction increases with the mass, although variations in stiffness and internal damping may in cases of marked variation in these properties mask the effect of mass alone. Sound insulation is a problem of structure rather than of materials.

In his paper before this Institute, Professor Sabine very concisely stated the conditions for good acoustics in audience rooms, a statement which cannot be improved upon today. I quote it: "In order that hearing may be good in any auditorium, it is necessary that the sound should be sufficiently loud, that the simultaneous components of a complex sound should maintain their proper relative intensities, and that the successive sounds in rapidly moving articulation, either of speech or music should be clear and distinct, free from each other and from extraneous noises. These three are the necessary, as they are the entirely sufficient conditions for good hearing. As an engineering problem it involves the shape of the auditorium, its dimensions and the materials of which it is composed."

As illustrating the effect of shape we shall consider a particular shape that experience has uniformly shown to be acoustically troublesome, namely, the circular or nearly circular plan surmounted by a hemispherical dome. You will immediately recognize in this description a room which encloses a planetarium. The difficulty here is due to reflected sound which is focussed sharply by the curved walls and ceilings, resulting in highly concentrated echoes. By echo we mean the distinctly audible repetition of a sound of short duration produced by reflection from a surface so located that there is a perceptible interval between the direct and reflected sounds. When, due to the shape of the reflecting surface,

FIG. 4.



Section of Fels Planetarium. The sky dome of perforated metal is almost perfectly transparent to the sound which is absorbed by mineral wool mounted on the external walls and ceilings.

the intensity of the reflected sound approaches that of the direct sound, the echo becomes annoying. By concentrating the sound which they reflect, extended concave surfaces produce troublesome echoes.

Thus the planetarium presents the problem of focussed echo in its most exaggerated form. The dome upon which the heavenly bodies are imaged is spherical, and the center of curvature of the sphere is in the center of the room, only a few feet above the floor. If this surface is acoustically reflecting in any considerable degree, then the sound originating near the floor will after reflection from the dome be focussed at one or more points in the room. Therefore the planetarium dome must be of a material that is highly reflecting, optically, and at the same time acoustically very transparent. In the earlier planetaria, this end was achieved by using an unsized fabric through which a large percentage of sound is transmitted.

The practical difficulties of securing a true spherical surface with such a material and also of renewing the surface when as inevitably happens it accumulates dirt, make this solution unsatisfactory. In certain cases, too, the builders neglected to consider the question as to what is to become of the sound after it gets through the dome. In one instance of which I know, the sky dome is surmounted by a second dome, which returns the focussed sound through the muslin sky, producing conditions not much better than if the sound were reflected from the sky dome itself.

In the Fels Planetarium, a solution for the problem was found in the use of a perforated metal dome. At first glance it would appear that a degree of perforation that would render the metal acoustically transparent would destroy its usefulness as an optically reflecting surface. As a matter of fact it was found that, with the proper size and spacing of holes and with metal not too thick, a dome could be constructed that would be optically highly reflecting and at the same time almost perfectly transparent acoustically. Laboratory tests showed that a metal surface with the proper size and spacing of holes and in which the perforations were only 1/8 of the total area could transmit 97 per cent. of the sound which strikes it. In this respect it was considerably better than unsized muslin. Thus only 3 per cent. of the sound striking the sky dome would undergo focussing reflection.

The problem is not solved, however, until we employ some means for dissipating this transmitted sound. The obvious thing to do is to absorb as large a fraction of it as possible, and this was done by lining the walls and the ceiling of the space above the dome with highly absorbent material. In order to prevent focussing of the small fraction not absorbed by these surfaces from being concentrated, the ceiling and the side walls above the dome were made plane rather than curved so as to avoid any concentration of the sound which must find its way back to the audience through the perforated metal dome.

The question naturally arises as to whether the same purpose might not have been perfectly well served by mounting the absorbent material directly back of the dome. This question, I think, must be answered in the negative, but not without experimental evidence to support this conclusion.

Actual measurements show that between 80 and 90 per cent. absorption can be obtained when a highly absorbent material is mounted directly back of perforated metal similar to that used in the Fels Planetarium dome. This means that instead of the three per cent. of focussed reflection from the dome, we should have from 10 to 20 per cent. focussed reflection if we mount our absorbent material directly back of the spherical surface. In other words, we should have roughly three to six times as much focussed reflection as if we mounted the absorbent material on walls and ceiling away from the dome. In the latter case we shall have possibly the same amount of sound returned, but with the important difference that only the three per cent. reflected directly from the perforated spherical surface will be subject to the focussing action.

The securing of good hearing conditions does not impose any very hard and fast demands that must be met at the sacrifice of other desirable features of design. Rooms are rendered acoustically good through the absence of defects rather than by the possession of positive virtues. As evidence of this may be cited the fact that in the older theaters of conventional types, with two and sometimes three balconies and seating less than 1200 people, the acoustic properties are generally quite excellent. As exceptions, one calls to mind certain outstanding examples in which in response to the client's demand for something novel and striking, the architect has resorted to new and untried forms, with unfortunate acoustical results. Limitations as to size, the necessity of keeping distances from the stage small enough that the most remote seats will come within the range of the unaided speaking voice, tend to obviate acoustical defects.

With the advent of talking motion pictures, and of satisfactory means of sound amplification, these limitations on the size of audience rooms have been removed. The present trend toward much larger houses, intended for widely diversified uses, including vaudeville numbers, sound films, and orchestral performances, makes acoustics a much more important element of the designer's problem. Viewed from the standpoint of utility only, this problem is simply to provide seats for a large number of people so that all may see and hear clearly what transpires on the stage.

There are a few simple rules which may be followed in design that will go far towards achieving this result. They may be enumerated as follows:

1. Side walls and ceiling in the front of the room should reflect the sound at nearly glancing angles to the sides and rear of the room.

2. Extended curved wall or ceiling surfaces, particularly those with curvature in two planes, should not have centers or axes of curvatures that fall either on the stage or near any portion of the audience.

3. The average ceiling height should be determined by the number of seats, so that the cubic contents per seat is not greater than about 200.

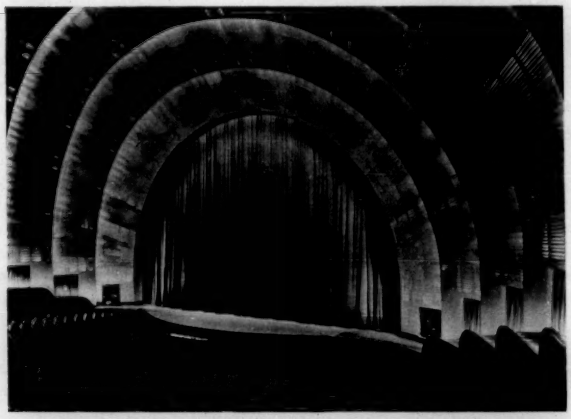
4. The total sound-absorbing power should be such that with the average audience present the reverberation time computed by the reveration formula will fall within the range of acceptable values for a room having the volume of the proposed room.

We might illustrate these principles of acoustical design by reference to numerous examples in which acoustical planning has led to entirely satisfactory results. It will perhaps serve our purpose quite as well if we content ourselves with a single outstanding case. I refer to the large new Music Hall of the Rockefeller Center popularly known as Radio City. Here the matter of acoustics was of paramount importance and the problem was complicated by the huge size and the wide variety of uses to which the room was to be put. The program called for a seating capacity of 6200 and for good hearing conditions for entertainments ranging from vaudeville sketches to super-stage spectacles and large orchestral performances, with talking motion pictures and organ recitals thrown in for good measure. Added to this was the provision of equipment for lighting effects throughout the entire hall that would surpass anything hitherto achieved. In fact everything was to be on a super-scale.

The modern spirit of the building as a whole dictated a minimum of architectural embellishment of the interior. The requirement of 6200 seats called for large plan dimensions, which architecturally called for a great ceiling height and a huge volume. The dimensions are roughly 160 feet from

curtain line to rear wall, by 176 feet wide at the rear with an average ceiling height of 80 feet, giving a total volume of some 1,800,000 cubic feet, almost 300 cubic feet per person. This combination of plain treatment and large volume would normally lead to excessive reverberation, since reverberation is directly proportional to volume and inversely proportional to the total absorbing power. Moreover, both structural and architectural necessities dictated curved contours, from which, without prevision, troublesome focussed echoes might well be

FIG. 5.



Interior of the large Music Hall of Rockefeller Center, New York.

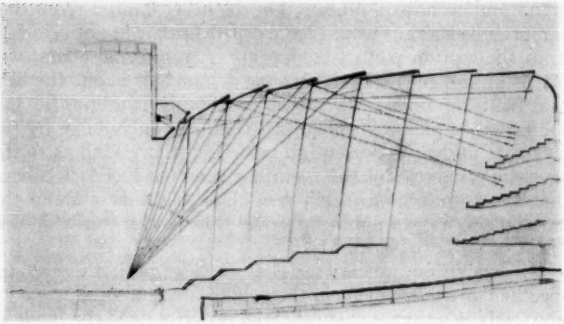
expected. The traditional solution of the lighting problem is light coves at the springline of a ceiling dome. Here again would be a fruitful source of acoustical difficulties. Finally the great width, and the extreme distance to the rear call for whatever reinforcement by reflection is possible for seats at the side and in the rear of the hall.

Looking toward the stage we see a series of expanding arches in which the stage is centered. The effect is one of architectural simplicity with only very low relief ornamentation. Back of the grilled openings is a portion of the lighting

equipment from which marvellously beautiful effects result. The overlapping of the arches provides ample space for further concealed lighting. The light thus seems to come from everywhere in general and no place in particular.

Viewed in plan, the arches overlap like the bands of a huge armadillo. While presenting the effect of a plane surface from the interior, these architectural breaks are sufficient to keep the ceiling and side walls from acting like a vast acoustical mirror. In longitudinal section, we note how the disposal of the arches is such that sound from the stage striking these surfaces at almost glancing incidence is directed not downward

FIG. 6.



Longitudinal section of Music Hall showing reflection of sound at glancing incidence from the ceiling arches to rear balcony seats.

where it would cause an annoying ceiling echo, but toward the seats in the two upper balconies. A similar construction in plan would show the sound directed to the rear and sides of the room. In short, the contours in both plan and section are designed to take advantage of reflection from the front portions of the room so as to give as uniform a distribution of sound intensity as possible. The result is that for an auditor seated in the extreme rear of the upper balcony sound of even conversational loudness comes from the stage with remarkable clarity.

Finally in order to prevent the excessive reverberation which would materially lessen the intelligibility of speech, a

sound absorbing plaster was specified for the entire area of the ceiling arches. Plastered in ordinary material these surfaces would absorb only three or four per cent. of the sound striking them, and the resulting reverberation time, would have been something over 2.5 seconds. The sound absorbing plaster used has a coefficient of about 28 per cent. and the computed reverberation time interpolated is about 1.9 seconds. The time 1.9 seconds for this hall with its volume of 1,800,000 cubic feet would fall near the middle of the range of acceptable times for all uses, a compromise that seemed wise in view of the varied forms of entertainment that were anticipated. It is quite possible that organist and orchestral leaders will complain that the room is too dead, i.e., too little reverberant for organ and orchestral music. On the other hand a room that is more reverberant would have called forth a fire of criticism from the motion picture engineers. A compromise that will placate if not please both extreme demands is about the best that can be hoped for. How well this compromise has succeeded in pleasing public taste one can only judge by the fact that up to the present no criticism of any kind as to the acoustic properties of the room has been received. I believe that the architect ordinarily considers that this is about the most in the way of public approval that he can expect in this "best of all possible worlds."

In conclusion, we may say that while we cannot report any spectacular achievement in the field of architectural acoustics since Sabine's time, yet substantial progress has been made in the practical application of the scientific principles which he enunciated. Well authenticated data on sound insulation are now available. A wide variety of materials that have excellent sound absorptive properties and which have desirable structural properties have been developed to meet almost any desired mode of auditorium treatment, in place of the unsightly hair felt of the earlier days. The new developments in the recording and reproduction of sound have given us both new problems and new tools with which to solve them. The wilderness of guesswork and uncertainty into which Sabine as a pioneer plunged forty years ago, is today a region of cultivated and well ordered scientific knowledge. Architectural acoustics may well lay claim to being an established and self respecting branch of engineering science.

THE COEFFICIENT OF RESISTANCE AS A FUNCTION OF REYNOLDS NUMBER FOR SOLIDS OF VARIOUS SHAPES.

BY

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University of Chicago.

ABSTRACT.

The purpose of the present article is twofold. First it is an attempt to show the usefulness of the degree of true sphericity as an expression for shape of rock particles. Second it is intended as a background for a new, analytical, sedimentographic method to be given in a forthcoming paper. Any radical variation in shape influences the numerical value of the degree of true sphericity and affects the size of the surface area, the size of the largest cross-sectional area and the degree of circularity of a great number of cross-sections of a solid. These factors determine to a large extent the settling velocity of that solid. The resistance of wholly submerged bodies, as a theoretical foundation for determining the coefficient of resistance as a function of Reynolds number for solids of different degree of sphericity, is discussed. The influence of the degree of circularity on the resistance is then shown by graphs constructed on the basis of available data. Formulæ for calculating the coefficient of resistance and Reynolds number are introduced and their practical value demonstrated by graphic illustrations. The sedimentological significance and usefulness of the results are discussed.

Index of Symbols and Conceptions Used in this Paper.

A = the cross-sectional area of a pipe, or a solid's cross-sectional area normal to the fluid motion.

C = the length of the circumference or perimeter of a plane figure of any shape.

c = the length of the circumference of a circle area.

$\acute{\epsilon}$ = the degree of circularity c/C, i.e. the ratio of the length of the circumference of a circle area equal to the area of a given plane figure, to the actual perimeter of the plane figure.

C_R = the coefficient of resistance.

d = the diameter of a sphere or the diameter of a pipe of circular cross-section.

d_n = the true nominal diameter of a solid, i.e. the diameter of a sphere of the same volume as the solid, irrespective of the latter's shape.

f = a mathematical symbol meaning function.

g = the acceleration of gravity the numerical value of which is taken as 981 (cm. sec.$^{-2}$) for all calculations in this paper.

l = any length defining the scale of a body.

M = the mass.

m = the amount of magnification; for instance if a straight line of 1 cm. in length is magnified to 4 cm., then m equals 4.

μ = the coefficient of viscosity.

ν = the kinematic viscosity.

r_n = the true nominal radius, i.e. the radius of a sphere of a volume equal to that of the solid, irrespective of the latter's shape.

ρ_S = the specific gravity of a solid.

ρ_F = the specific gravity of a fluid.

$\mathbf{R}$ = a non-dimensional expression called *Reynolds number* obtained by the formula $vd_n\rho_F/\mu = vd_n/\nu$.

R = the resistance.

s = the surface area of a sphere.

S = the surface area of a solid of any shape.

ψ = the degree of true sphericity obtained by the formula $\psi = s/S$.

V = the volume of a solid.

v = the velocity of a solid in translation, or the velocity of fluid motion.

volume, surface area and length are expressed in cubic centimeters, square centimeter and centimeter respectively.

THE DIAMETER, THE SURFACE AREA AND THE LARGEST CROSS-SECTIONAL AREA OF A SOLID.

Various opinions exist as to the meaning of "size" and "mean diameter" of a particle, and "average size" and "average diameter" of non-uniform granular substance. Attention has been given these questions notably by J. C.

Pearson and W. H. Sligh,[1] G. J. Perrott and S. P. Kinney,[2] G. Martin,[3] H. Green,[4] L. T. Work,[5] K. Kasai [6] and Th. Hatch.[7]

The term *diameter* is derived from the Greek δία through, and μετρον measure. The Oxford English Dictionary [8] gives a rather exhaustive information about the use of the term. Green (op. cit.) adopted the general definition. He said:

"The diameter of a particle is any straight line passing through the centre of the particle and terminated at the particle boundary. If the particle is a sphere, the length of its diameter is the same in all directions and consequently 'diameter' defines 'size' and the two terms become synonymous. For any shape other than the sphere 'diameter' ('size') must be carefully defined."

The general definition defines the *genus* diameter only.

[1] J. C. Pearson and W. H. Sligh, "An Air Analyzer for Determining the Fineness of Cement," *Techn. Papers Bureau of Standards*, No. 48, 1–74, 1915.

[2] G. J. Perrott and S. P. Kinney, "The Meaning and Microscopic Measurement of Average Particle Size," *Jour. Am. Ceramic Soc.*, **VI**, 417–439, 1923.

[3] G. Martin, "Law Governing the Connection between the Number of Particles and Their Diameters in Grinding Crushed Sand," *Trans. Ceramic Soc.*, **XXIII**, 61–120, 1923–24.

G. Martin, "Connection between the Statistical Diameter and the Statistical Volume of Irregular Shaped Particles of Crushed Sand," *Trans. Ceramic Soc.*, **XXVII**, 247–258, 1927–28.

G. Martin, "On the Connection between the Statistical Diameter of Crushed Sand Particles and Their Statistical Surface," *Trans. Ceramic Soc.*, **XXVII**, 259–284, 1927–28.

G. Martin, "Calculations Relating to Diameters, Surface, and Weights of Homogeneous Grades of Crushed Quartz Sand," *Trans. Ceramic Soc.*, **XXVII**, 285–289, 1927–28.

[4] H. Green, "A Photomicrographic Method for the Determination of Particle Size of Paint and Rubber Pigments," JOUR. FRANKLIN INSTITUTE, **CXCII**, 637–666, 1921.

H. Green, "The Effect of Non-Uniformity and Particle Shape on the 'Average Particle Size,'" JOUR. FRANKLIN INSTITUTE, **CCIV**, 713–729, 1927.

[5] L. T. Work, "Methods of Particle Size Determination," *Am. Soc. for Testing Materials, Proceedings*, **XXVIII**, Part 2, 771–812, 1928.

[6] Kōichi Kasai, "Über die Bestimmung der Teilchengrösse pulverförmiger Substanzen," *Scientific Papers of the Institute of Physical and Chemical Research*, **XIII**, pp. 135–183, Tokyo, 1930.

[7] Theodore Hatch, "Determination of 'Average Particle Size' from the Screen-Analysis of Non-Uniform Particulate Substances," JOUR. FRANKLIN INSTITUTE, **CCXV**, 27–37, 1933.

[8] James A. H. Murray, "A New English Dictionary," Oxford, 1914.

Dealing with non-spherical solids further specifications are needed in respect to the length and position of the diameter within the solid. The diameter of a sphere has a *proprium*, namely, that its length equals $6V/s$, where V is the volume and s the surface area of the sphere. Since all diameters in a sphere have the same length, the "mean diameter" of a sphere equals the volume-surface ratio multiplied with 6. The coefficient (6) is characteristic for a sphere and cannot be adopted for any other shape, unless similarity of volume and surface area is disregarded. Green (op. cit.), however, considered the length of the diameter obtained by the formula above as a kind of "natural one," because it holds for a cube, if the length of the diameter of this shape is taken as equal to that of the edge. Thus the cube and the inscribed sphere have the same "mean diameter," which is not true, because the diameter equal in length to the edge of the cube represents the smallest diameter. Furthermore, the cube has a volume and surface area almost double (1.9 times) those of the inscribed sphere. It is evident that in this case the "mean diameter" as an expression for "size" will not serve, if we wish to make any fundamental investigation in respect to physical properties of a solid and their influence on physico-chemical processes.

Let l be the length of a straight line of arbitrary position within a solid of any shape, for instance the diameter of a sphere, the edge of a cube, or the space diagonal of a parallelopiped. Let S and V be the surface area and the volume respectively. Then, if l is magnified m times and geometric similarity of the body is maintained, the enlarged particle has a surface area Sm^2, and a volume equal to Vm^3. Thus it makes no difference which line we choose for expressing "size" as long as geometric similarity is maintained. Since after all the size of the volume and the surface area (and the specific gravity) are the important properties of a solid, why should we then confuse these simple conceptions by introducing a "mean diameter" as an expression for "size"? Except for a sphere, the size of the "mean diameter" cannot be used for calculating the size of the volume nor the size of the surface area accurately.

The diameter of a sphere of the same volume as the par-

ticle may sometimes be advantageously used in mathematical operations. This "diameter" was in a previous paper [9] termed the *true nominal diameter*. The term is new [10] but the idea has previously been expressed in more or less the same form by Owens,[11] Bölsterli,[12] Schulz,[13] Work,[14] Prockat [15] and probably others. Owens and Bölsterli expressed the grain "size" by the diameter of a sphere equal in *weight* to the particle. Work introduced the term *mean spherical diameter* for the diameter of a sphere of a weight equal to the average weight of a number of particles. Work's term is rather expressive and therefore practical, although some may object to the use of *spherical* in connection with *diameter*. Work's term, however, refers to a sphere of equivalent weight, while in the present study equivalent volume is the essential. Schulz introduced the term "ideeller Durchmesser" (ideal diameter) for the diameter of a sphere of the same volume as the particle. Thus Schulz's term is synonymous with my term *true nominal diameter* and has the precedence of the latter in respect to time of introduction. When I first suggested the term "true nominal diameter" (op. cit.), Schulz's work was not available to me and it was first later by the courtesy and efforts of the Crerar Library of Chicago that his dissertation became ac-

[9] Hakon Wadell, "Volume, Shape and Roundness of Rock Particles," *Journal of Geology*, **XL**, 444, 1932.

[10] The term is not entirely new. *Nominal diameter* (*a*) is applied by mechanical engineers as an arbitrary term for the inner diameter of a pipe, but, being used in a somewhat different field of endeavor, there is no risk of confusing it with the term *true nominal diameter* as used in this paper.

(*a*) J. C. Trautwine, "The Civil Engineer's Pocket-Book," 1919, p. 526.

[11] J. S. Owens, "Experiments on the Settlement of Solids in Water," *Geographical Journal*, **XXXVII**, 59–79, 1911.

[12] A. Ludin, "Die Wasserkräfte"; in respect to Bölsterli see Vol. II, 1913, p. 899.

[13] P. Schulz, "Neue Bestimmungen der Konstanten der Fallgesetze in der nassen Aufbereitung mit Hilfe der Kinematographie und Betrachtungen über das Gleichfälligkeitsgesetz," *Von der K. Sächs. Technischen Hochschule zu Dresden in Verbindung mit der K. Sächs. Bergakademie zu Freiberg zur Erlangung der Würde eines Doktor-Ingenieurs genehmigte Dissertation*, 1914, p. ix.

[14] Op. cit., p. 784.

[15] F. Prockat, "Beitrag zur Kenntnis der Vorgänge bei der trockenen Aufbereitung und Abscheidung von Stauben," *Dissertation eines Doktor-Ingenieurs. D. Techn. Hochschule zu Berlin*, 1929, p. 19.

cessible. Since, however, Schulz failed to show the entire usefulness of the "ideal diameter" I have felt justified to maintain the expression *true nominal diameter*. The Oxford English Dictionary gives the following interpretation of the word *nominal*. "4. Existing in name only, in distinction to *real* or *actual;* merely named, stated, or expressed, without reference to reality or fact." Since the diameter of a sphere of the same volume as the particle is not any real or actual diameter of the particle, the term *true nominal diameter* seems to be justified.

If a sphere is deformed, the length of its diameters in different positions within the solid are changed, some shortened, others lengthened. Thus the change in shape effects the length of the diameters. For the sake of theoretical simplicity assume that we are dealing with a cube, the diameters of which are a, b and c, each equal in length to the edge of the solid. This is not strictly correct in view of our previous discussion, but the example is chosen in order to simplify the illustration that the diameters are influenced by the shape. It is evident that $a \times b \times c$ equals the volume of the cube. If we agree that the change in shape involves no change in volume, then the change of the surface area is the principal factor in shape modifications. Call a the *longest diameter*, b the *intermediate diameter* and c the *shortest diameter*. In the case of a cube they are of the same length (for the purpose of our discussion). If now a in the cube is magnified m times, then b and c, or c alone must be shortened proportionally in order that the solid shall maintain the same volume. Table I illustrates four cases. When the cube is deformed to a parallelopiped of the type II, the diameter a is magnified m times to a length of am, and b and c are reduced in size to $b/\sqrt{m}$ and $c/\sqrt{m}$ respectively. It is evident that no change in volume takes place because $am \times b/\sqrt{m} \times c/\sqrt{m} = a \times b \times c$ = volume of the original cube. By these changes in the length of the diameters, the surface area of the cube is also increased according to the formulæ in Table I. In other words the degree of sphericity is decreased from the cube to parallelopiped II. Another important factor in the present study is the change of the largest cross-sectional area. Again for the sake of theoretical simplicity, assume that the largest

cross-sectional area of the cube equals $a \times b$, i.e. it equals one of the plane faces of the cube. Then parallelopiped II will have a "largest" cross-sectional area equal to $am \times b/\sqrt{m} = \sqrt{m}$, because a and b are units of the same magnitude. Substituting numerical values will clarify the problem. Assume that we have a cube of an edge equal to 1 cm. Thus a, b, and c are each equal to 1 cm. The volume of the cube equals $a \times b \times c = 1 \times 1 \times 1 = 1$ cubic centimeter. Assume further that a is magnified 4 times, i.e. $m = 4$. We have then for parallelopiped II:

"Longest" diameter $= am = 1 \times 4 = 4$.

"Intermediate" diameter $= \frac{b}{\sqrt{m}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$.

"Shortest" diameter $= \frac{c}{\sqrt{m}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$.

Volume $= am \times \frac{b}{\sqrt{m}} \times \frac{c}{\sqrt{m}} = 4 \times \frac{1}{2} \times \frac{1}{2} = 1$ = volume of the original cube.

Surface area $2\left(2\sqrt{m} + \frac{1}{m}\right) = 2\left(2\sqrt{4} + \frac{1}{4}\right) = 8\frac{1}{2}$ (Surface area of the original cube equals 6).

"Largest" cross-sectional area $am \times \frac{b}{\sqrt{m}} = \sqrt{m} = \sqrt{4} = 2$

(The "largest" cross-sectional area of the cube is $1 \times 1 = 1$).

If we follow the same method of calculation for parallelopipeds III and IV, the result which may be read from Table I can be summarized as follows:

1. Change in shape of a solid involves change in length of its diameters. The arithmetic mean of the three diameters increases with increasing surface area. The geometric mean of the diameters remains, at least theoretically, the same.[16]

[16] Though in the theoretical case above the volume of a particle may be calculated by the geometric mean of its three diameters, the practical result is not guaranteed as long as the meaning of the longest, intermediate and shortest diameters of irregular shapes remains non-defined.

TABLE I.

The Relationship between the Diameters, the Surface Area and the "Largest" Cross-sectional Area of Rectangular Parallelopipeds.

	Cube I		Parallelopiped II		Parallelopiped III		Parallelopiped IV
"Longest" diameter	$a = 1$		am		am		am
"Intermediate" diameter	$b = 1$		$\frac{b}{\sqrt{m}}$		b		bm
"Shortest" diameter	$c = 1$		$\frac{c}{\sqrt{m}}$		$\frac{c}{m}$		$\frac{c}{m^2}$
Volume	$a \times b \times c = 1$	$=$	$am \times \frac{b}{\sqrt{m}} \times \frac{c}{\sqrt{m}} = 1$	$=$	$am \times b \times \frac{c}{m} = 1$	$=$	$am \times bm \times \frac{c}{m^2} = 1$
Surface area	$2(ab+ac+bc) = 2 \times 3$	$<$	$2\left(2\sqrt{m}+\frac{1}{m}\right)$	$<$	$2\left(m+\frac{1}{m}+1\right)$	$<$	$2\left(m^2+\frac{2}{m}\right)$
"Largest" cross-sectional area	$ab = 1$	$<$	$\sqrt{m}$	$<$	m	$<$	m^2

2. The surface area of the solids increases from left to right in the table, in other words, the degree of sphericity decreases from a cube towards parallopiped IV. The "largest" cross-sectional area increases with increasing surface area or with decreasing degree of sphericity.

If we take m as a constant in Table I, and construct the four types of geometric solids on a piece of paper, we would in a popular terminology refer to the cube as "equidimensional," or if its corners were somewhat rounded off so as the form approached the shape of a sphere, this form would, in a somewhat incorrect but commonly adopted manner, be called "roundish." Parallelopiped II would be called a "long and narrow" solid, parallelopiped III a "long and somewhat flat" particle, and parallelopiped IV would decidedly be called a "flat" solid since am and bm are large and the "shortest" diameter c/m^2 is very small. It may here be appropriate to call the attention to Richards' experience on settling velocities of ore particles. He said:[17]

"Of particles which just pass through the same screen, the roundish grain settles faster than the long, narrow grain, and the latter settles faster than the flat grain."

THE RESISTANCE DUE TO FLUID MOTION ABOUT WHOLLY SUBMERGED BODIES.

A body in steady motion through any real fluid, or at rest in a current, exhibits a resistance consisting in part of frictional forces tangential to the surface of the body, in part of pressure forces acting at right angles to the surface. If the speed is constant, so that there is no acceleration except at the start of motion, and if the fluid is incompressible, or if in a compressible fluid the speed does not approach the acoustic speed so that the pressure changes are so small that the compressibility may be neglected, the resistance may depend mainly upon the relative velocity of fluid and body, on the density and viscosity of the fluid, on the volume, specific gravity and shape of the body, and also upon the surface roughness of the solid. Sedimentology is concerned with the settling velocities of rock particles in air and water. These media can be considered as incompressible, or the

[17] R. H. Richards, C. E. Locke and J. L. Bray, "Ore Dressing," 1925, p. 127.

influence of compressibility is negligible on account of the relatively slow motions involved in ordinary processes of sedimentation.

Motion of real fluids, as distinguished from the "perfect fluids" of the mathematician, is accompanied by internal friction which resists the relative movements. This is generally referred to as the viscous resistance. Newton [18] gave a simple theoretical illustration of the phenomena. The definition of viscosity by Maxwell is as follows: [19]

"The viscosity of a substance is measured by the tangential force on the unit of area of either of two horizontal planes at the unit of distance apart, one of which is fixed, while the other moves with the unit of velocity, the space between being filled with viscous substance."

The theory of viscosity is best conceived in the case of laminar motion, which can be regarded as a series of very thin layers of fluid, sliding upon each other with a constant small velocity in a common direction over a fixed, smooth plane surface. The fluid in contact with the plane surface adheres to it and will therefore be at rest. The horizontal velocity of the successive overlaying laminæ is proportional to the velocity gradient v/y, v being the horizontal velocity at a perpendicular distance y above the fixed plane surface. Assuming, according to Newton, that the tangential stress between the laminæ is proportional to the velocity gradient, the relation between the internal friction and the relative motion is then given by the equation,

$$F = \mu S \frac{\partial v}{\partial y}, \tag{1}$$

where F is the internal friction, or the tangential force opposing the relative motion, S is the area of the fixed plane surface, ∂v and ∂y the differentials of v and y respectively, and μ is the *coefficient of viscosity*, often simply called the *viscosity*, or the *absolute viscosity*, and sometimes the *dynamic viscosity* [20] as

[18] Isaac Newton, "Philosophiae Naturalis Principia Mathematica," Liber II, Sectio IX, Editio tertia aucta & emendata, 1726. (Reprinted 1871 for W. Thompson and H. Blackburn, pp. 374–383.)

[19] J. Clerk Maxwell, "Theory of Heat," 1872, p. 278.

[20] S. Erk, "Zähigkeitsmessungen," *Hydro- und Aerodynamik*, Handbuch der Experimentalphysik (Ed. by W. Wien and F. Harms), Bd. IV, Teil 4, 1932. p. 463.

distinguished from the *kinematic viscosity*. Thus the internal friction, or viscous resistance, is proportional to the surface area, to the velocity gradient and to a constant μ which differs with each kind of fluid.

Osborne Reynolds [21] found by a number of experiments that, the more viscous the liquid, the less prone is it to eddying or sinuous motion. He also found that the eddying began at a definite value of $v \times d$, where v is the velocity and d the diameter of the pipe. The quantity which determines the type of flow is, however, not the viscosity coefficient, but the viscosity coefficient divided by the density μ/ρ_F,—a constant which by Maxwell [22] was called the *kinematic viscosity*, generally denoted by the Greek letter ν.

It was also shown by Reynolds that the conditions of flow in any pipe and for any liquid could be characterized by a pure number, or numeric. The variables affecting the flow in a cylindric pipe can be combined to a non-dimensional expression:

$$\mathbf{R} = \frac{vd\rho_F}{\mu} = \frac{vd}{\nu}, \tag{2}$$

in which **R** represents the *Reynolds number*, v is the mean velocity, d the diameter of the pipe, and ν is the kinematic viscosity μ/ρ_F. From the dimensional point of view, the flow past fixed obstacles or, a completely immersed body moving through a fluid, is dealt with in the same way as that of the pipe. The diameter of a sphere, for instance, takes the place of the diameter of the pipe, and the translation velocity of the sphere takes the place of the mean velocity of the fluid in the pipe. Dynamic simularity is generally assumed to exist between all cases for which Reynolds number is a constant.

At low Reynolds numbers (**R**-values) when the flow is laminar, the resistance is due essentially to the viscosity. At higher **R**-values, when the motion is definitely turbulent, i.e. at velocities above a critical value, the resistance is always in

[21] Osborne Reynolds, "An Experimental Investigation of the Circumstances which determine whether the Motion of Water shall be Direct or Sinuous, and of the Law of Resistance in Parallel Channels," *Phil. Trans. Royal Soc. London*, **CLXXIV**, 935–982, 1883.

[22] Op. cit., pp. 278–279.

some part due to the viscosity, but mainly to eddy formation. For larger **R**-values the eddy is not in stationary position, but a series of eddies is found, each drifting off in the "wake" of the solid.

When the velocity of a solid, moving uniformly in a fluid, becomes so great that eddying motion is set up, we have very few means of determining theoretically the way the resistance varies.

The "quadratic law" of resistance as proposed by Newton reads:[23]

" . . . I say, that the greater parts of the systems are resisted in a ratio compounded of the duplicate ratio of their velocities, and the duplicate ratio of their diameters, and the simple ratio of the density of the parts of the system."

Newton's law of resistance may be expressed by:

$$R \propto v^2 l^2 \rho_F, \qquad \text{or} \qquad R \propto v^2 A \rho_F, \tag{3}$$

where R is the resistance, v the velocity, ρ_F the density of the fluid, and l any length defining the scale of the body. A cross-sectional area A at right angle to the fluid motion may take the place of l^2.

The theory upon which Newton founded his law of resistance has in some respects been proved to be untenable. Experimental results have shown that the resistance is mainly due to the pressure difference and frictional forces arising from the fluid motion about the solid. The older hypothesis took into consideration only the solid's front shape (opposing the motion), whereas, according to modern views, the total shape of the body, and often especially the form on the back (on the lee-side in respect to the flow of the liquid) determines the resistance (as far as the shape of the solid is concerned). The pressure acting, for instance, on a sphere in front (opposing the motion) in the case of turbulent conditions is greater, and the pressure on the back is less than when the solid is at rest. The resultant is equal to the force, or resistance, opposing the motion.

Nevertheless, Newton's law has in several cases been found in accord with experience. A solid in translation must force aside per second a mass of fluid $M = vA\rho_F$, thereby conveying

[23] Isaac Newton, "The Mathematical Principles of Natural Philosophy," translated by Andrew Motte, Vol. II, 1819, p. 89.

to each mass element a velocity, which can be considered as proportional to the velocity of the solid. The resistance is proportional to the amount of motion per second, thus [24]

$$R \propto Mv = Av^2\rho_F. \tag{4}$$

In the English literature it is often the custom to express the total resistance, to the translation through a liquid, of similar bodies, of every shape, in corresponding directions by a formula of the type [25]

$$R = C_R A v^2 \rho_F = f(\mathbf{R}) A v^2 \rho_F, \tag{5}$$

where R is the resistance, C_R the coefficient of resistance, A a representative area of the solid, usually the maximum cross-section or else the maximum projection area opposed to the flow, v the translation velocity of the solid, or if at rest, the velocity of the fluid, ρ_F the density of the fluid, **R** Reynolds number, and f meaning function.

In Germany and Austria, and recently also in America and Italy, a formula of a slightly different type has been adopted. It reads:

$$R = C_R A \frac{v^2\rho_F}{2} = f(\mathbf{R}) A \frac{v^2\rho_F}{2}. \tag{6}$$

The letters denote the same as in formula no. 5. The velocity pressure, also called the "dynamic pressure" (Germ. Staudruck), is represented by $v^2\rho_F/2$, i.e. the pressure which would be exhibited per unit area on A in an "incompressible" fluid. The area A in the case of a sphere equals πr^2. Thus the coefficient of resistance in the case of a sphere and according to formula no. 6 is:

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[24] L. Prandtl, "Mechanik der flüssigen und gasförmigen Körper," *Müller-Pouillets Lehrbuch der Physik*, Edited by Erich Waetzmann, Bd. I, Teil 2, 1929, p. 1136.

[25] Horace Lamb, "Hydrodynamics," 1930, p. 644: ". . . we are led by consideration of dimensions to a formula of the type,

$$F = \rho U^2 l^2 f\left(\frac{Ul}{\nu}\right)."$$

In Lamb's formula U stands for the velocity, and l is any length defining the scale of the body.

$$C_R = \frac{R}{A\frac{v^2\rho_F}{2}} = \frac{R}{\pi r^2\frac{v^2\rho_F}{2}}. \tag{7}$$

When a solid has attained its terminal, constant settling velocity, the effective weight, i.e. the weight of the solid immersed in the fluid, is exactly balanced by the fluid resistance. We have then the well-known formulation for the resistance R:

$$R = V(\rho_S - \rho_F)g, \tag{8}$$

where R is the resistance, V the volume of the solid, ρ_S the specific gravity of the solid, ρ_F the specific gravity of the fluid, g the acceleration of gravity.

From the equations (7) and (8) we obtain the following formula for the coefficient of resistance,

$$C_R = \frac{2V(\rho_S - \rho_F)g}{Av^2\rho_F}. \tag{9}$$

From the equations (5) and (6) it follows that $C_R = f(\mathbf{R})$, i.e. the coefficient of resistance is a function of Reynolds number. The numerical value of C_R varies for non-spherical solids with the shape and position of the body with reference to its shape and the fluid motion or the direction of translation of the solid.

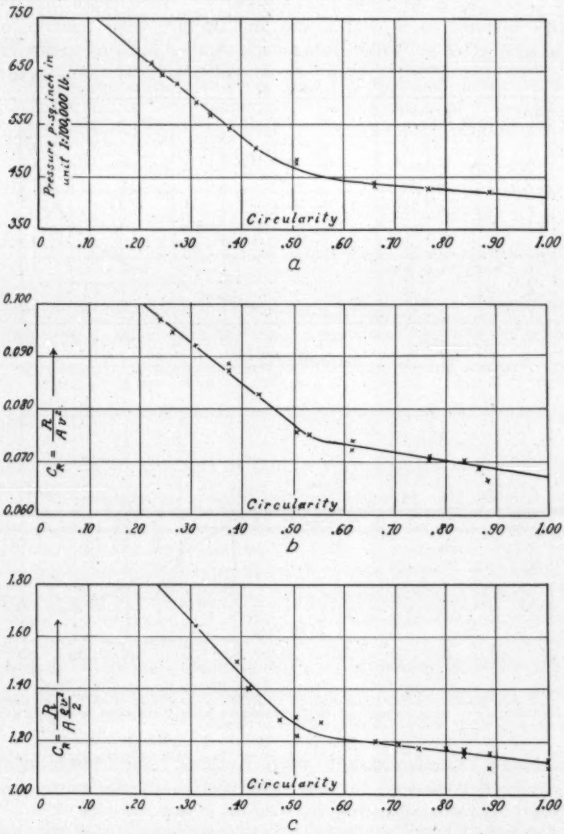
In Hydro- and Aerodynamics it is customary to represent experimental data graphically on a coördinate system, the **R**-values being laid off along the x-axis, and the values of the coefficient of resistance on the y-axis. Logarithmic charts are generally used. The lowest curve in Fig. 3 represents the results obtained for spheres by Wieselsberger,[26] Arnold,[27] and Allen.[28] **R**-values higher than 100,000 have been omitted. Wieselsberger's results were achieved by measurements in wind-tunnels in the Aerodynamic laboratory in Göttingen.

[26] C. Wieselsberger, A. Betz, L. Prandtl, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," Lieferung II, 1923, p. 29.

[27] H. D. Arnold, "Limitations Imposed by Slip and Inertia Terms upon Stokes's Law for the Motion of Spheres through Liquids," *Phil. Mag.*, 6th Ser., **XXII**, 762, 1911.

[28] H. S. Allen, "The Motion of a Sphere in a Viscous Fluid," *Phil. Mag.*, 5th Ser., **L**, 1900, Tables on pp. 334–337.

FIG. 1.

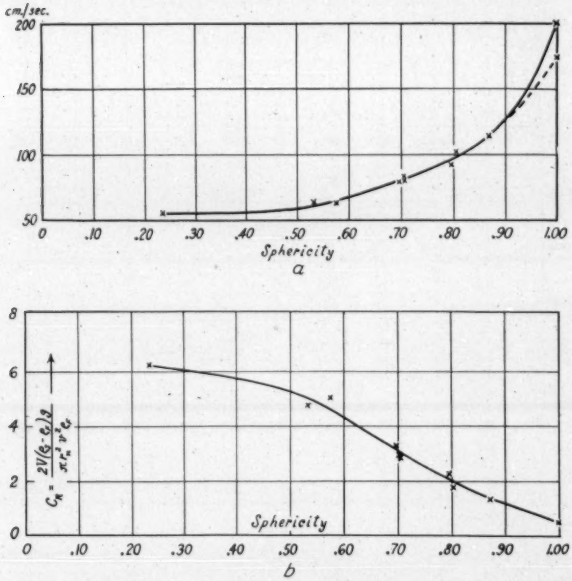


The resistance exhibited by plates of different plane shapes expressed in terms of degree of circularity.

From experimental data, *a*, by Stanton, *b*, by Eiffel, *c*, by Wieselsberger, Betz and Prandtl.

Arnold's data refer to settling velocities of Rose Metal spheres in Colza oil, a liquid of high viscosity. Allen experimented with airbubbles in water and in Aniline, with spheres of Paraffin Wax in Aniline, and with Amber spheres in water.

FIG. 2.



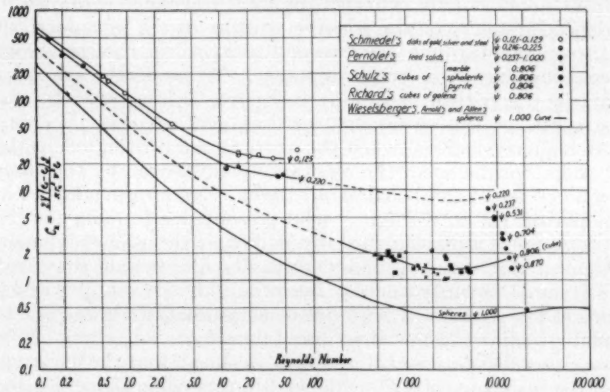
a, illustrating the influence of the degree of sphericity on the settling velocity of lead solids of approximately the same volume and weight. *b*, the coefficient of resistance as a function of the sphericity value. Based on experimental data by Pernolet.

Charts by Wieselsberger, Prandtl-Tietjens,[29] and L. Schiller [30] have been consulted for the construction of the curve for *spheres* represented in Fig. 3.

[29] O. Tietjens und L. Prandtl, "Hydro- und Aeromechanik," Bd. II, 1931, p. 131.

[30] L. Schiller, "Fallversuche mit Kugeln und Scheiben," *Hydro- und Aerodynamik*, Handbuch der Experimentalphysik (Ed. W. Wien und F. Harms), Bd. IV, 1932, Teil 2, p. 348 and p. 369.

FIG. 3.



The coefficient of resistance as a function of Reynolds number for solids of different sphericity values.

THE RESISTANCE OF PLATES OR LAMINÆ NORMAL TO THE FLUID MOTION.

The pressure on a solid in translation at velocities above the critical value, i.e. in the presence of eddying motion, is different at the head and at the tail. If the *solid shape* [31] of the head, as distinguished from the *plane shape* [32] of the front, is such that it would get the full dynamic pressure due to the velocity over its entire surface, the solid exhibits a maximum amount of resistance as far as the influence of its head is concerned. A plane or lamina with its broadside at right angle to the fluid motion fulfills this condition nearly, but not quite, as there is some loss of pressure at the margins.

Various theories for calculating the resistance of a blade or lamina normal to the flow have been advanced. The older

[31] The term *solid shape* is here understood to be synonymous with the solid geometric shape. For instance, the solid shape of the head of a sphere may be taken as synonymous with the geometric shape of a half sphere.

[32] The *plane shape* refers to the form of the cross-sectional area of the head normal to the flow of the liquid. The plane shape of the front of a sphere is circular, i.e. the cross-sectional area has a maximum degree of circularity.

theories due to Kirchhoff [33] and Rayleigh [34] considered the front pressure only, whereas the total resistance is known to depend not only on the positive pressure at the front face but also on the negative pressure or "suction" in the rear, both contributing to the total resistance. At moderate velocities above the critical value, the resistance is influenced by the strength of the stationary vortices generated in the rear. At higher velocities, when the vortices are drifting off in the "wake" of the solid, the resistance is influenced by the character of the so called *Kármánic vortex street*, a term adopted in honor of Th. v. Kármán,[35] who presented a formula for the resistance of a plate normal to the flow, in terms of the dimensions of the vortex system at some distance behind the plate. Kármán's formula has also been found to give results which are in fair agreement with direct measurements in respect to plates inclined to the direction of flow.[36]

Several experimental results have shown that the variation of the resistance of a plate depends on its plane shape, i.e. the shape of the plane parallel to the longest and intermediate diameters in the case of a position normal to the flow. The shape of rectangular plates is often expressed by the ratio l/w or w/l, l being the length and w the width of the plate. In Hydraulics [37] the conception 'profil radius' or 'hydraulic radius' [38] is expressed either by the ratio $2A/C$ or A/C, where A is the cross-sectional area of the pipe and C is its perimeter.

[33] G. Kirchhoff, "Zur Theorie freier Flüssigkeitsstrahlen," *Crelle's Journal f. d. reine und angewandte Mathematik*, Bd. LXX, 1869, pp. 289–298.

[34] Lord Rayleigh (J. W. Strutt), "On the Resistance of Fluids," *Phil. Mag.*, 5th Ser., **II**, 430–441, 1876.

[35] Th. v. Kármán, "Über den Mechanismus des Widerstandes, den ein bewegter Körper in einer Flüssigkeit erfährt," *Nachrichten v. d. K. Gesellschaft der Wissenschaften zu Göttingen, Math.-Phys. Klasse*, Jahre 1912, pp. 547–556.

Th. v. Kármán und H. Rubach, "Über den Mechanismus des Flüssigkeits- und Luftwiderstandes," *Physikalische Zeitschrift*, Bd. XIII, 1912, pp. 49–59.

[36] A. Fage and F. C. Johansen, "On the Flow of Air behind an Inclined Flat Plate of Infinite Span," *Proc. Royal Soc. London*, Ser. A, **CXVI**, 170–197, 1927.

[37] L. Schiller, "Strömung in Rohren," *Hydro- und Aerodynamik, Handbuch der Experimentalphysik* (Ed. W. Wien and F. Harms), Bd. IV, Teil 4, 1932, p. 140.

[38] The term *hydraulic radius* has an unfortunate resemblance to Schöne's term *hydraulic value* ("Körner gleichen hydraulischen Wertes"). See H. Gessner, "Die Schlämmanalyse," 1931, p. 11.

Both formulas are inadequate expressions for the cross-sectional form. It is readily shown that the ratio of the area to the perimeter is the same for the square and the inscribed circle, thus two distinctly different plane shapes having the same "profil radius." A formula which, appears to be an improvement, has been suggested by E. P. Cox,[39] however, on different grounds. His formula, for what he called roundness, reads:

$$K = \frac{4\pi A}{C^2},$$

where K is dependent upon the plane shape of the figure, being 1.00 for a circle and less than one for any other shape. A and C are the area and the perimeter respectively.

A formula, expressing the shape of a plane figure in terms of the degree of circularity, and based on the isoperimetric property of a circle, was suggested in one of my previous papers.[40] It reads:

$$\frac{c}{C} = \phi,$$

where c is the circumference of a circle of the same area as the plane figure, C the actual perimeter of the plane figure, and ϕ the degree of circularity. The maximum value obtained by this formula is 1.000 . . ., which is the numerical value for the degree of circularity of a circle.

Experimental results on the resistance of plates, achieved in the National Physical Laboratory of London,[41] and in the Paris [42] and Göttingen [43] aerodynamic laboratories, are graphically represented in Fig. 1, *a*, *b* and *c* respectively. The plane

[39] E. P. Cox, "A Method of Assigning Numerical and Percentage Values to the Degree of Roundness of Sand Grains," *Jour. Paleon.*, I, 179–183, 1927.

[40] Hakon Wadell, "Sphericity and Roundness of Rock Particles," *Jour. Geology*, **XLI**, 321, 1933.

[41] T. E. Stanton, "On the Resistance of Plane Surfaces in a Uniform Current of Air," *Minutes of Proceedings Inst. Civil Engineers*, **CLVI**, 110, 1904.

[42] G. Eiffel, "La Résistance de l'Air et l'Aviation," 2E Éd., 1911, p. 42.

[43] C. Wieselsberger, A. Betz, L. Prandtl, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," Lieferung II, 1923, p. 34.

L. Prandtl und A. Betz, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," Lieferung IV, 1932, p. 99.

shapes of the plates have been recalculated in terms of the degree of circularity according to the formula given above. The letters in the formulæ for the coefficient of resistance are in accord with the notations used in this paper. The general trends of the curves agree well in spite of the difference in the notations on the *y*-axis. Some uncertainty exists in respect to the trend of the curves between 0.80 and 0.90 degrees of circularity, both the French and German data showing some variations of results. I am not aware of any data referring to plates of very irregular outlines. There are reasons for assuming that slightly different results may be obtained in the presence of deep re-entrant angles.

The curves show a comparatively slow increase in the numerical value of the coefficient of resistance up to a circularity of about 0.52–0.56, where a notable change takes place with a more rapid increase in the numerical value of the coefficient of resistance for lower circularity values. This peculiarity is in all probability due mainly to alterations of the dimensions, character and strength of the vortex system in the rear with changes in the degree of circularity.

THE INFLUENCE OF THE SETTLING POSITION OF THE SOLID SHAPE WITH REFERENCE TO THE FLUID MOTION.

Franz Schulze [44] pointed out long ago that the translation velocity of a flat stone in air or water depends upon its position in respect to the direction of translation. Jordan [45] noted that mineral particles readily took a position with their broad side at right angle to the direction of fall. Lord Rayleigh [46] showed and proved by experiment that a lamina free to turn about its centre line sets itself across the direction of motion of the fluid. P. Schulz [47] observed that a cube had a tendency

[44] Franz Schulze, "Anleitung zur Untersuchung der Ackererden auf ihre wichtigsten physikalischen Eigenschaften und Bestandstheile," *Jour. f. prakt. Chemie* (Erdmann & Marchand), Bd. XLVII, 1849, p. 261.

[45] R. Jordan, "Bemerkungen über den freien Fall der Mineralkörner im Wasser mit Rücksicht auf Hundt's Stromsetzmaschine," *Zeitschrift für das Berg- und Hütten- und Salinenwesen in dem Preussischen Staate*, Bd. XIV, 1866, B. Abhandlungen, p. 200.

[46] Lord Rayleigh (J. W. Strutt), "On the Resistance of Fluids," *Phil. Mag.*, 5th ser., **II**, 430–41, 1876.

[47] Op. cit., p. 36.

to settle with one of its flat faces downwards and that this settling position was more readily taken by cubes of small volume and weight. Miyagi [48] found that air bubbles undergo a regular change in shape when rising in water and that the major axis of any bubble was always perpendicular to its course, that is, it moved always with its flattened face ahead. Schmiedel [49] found by experiment that thin disks at very low Reynolds number showed no tendency to take any particular settling position, while with increasing **R**-values up to 80 (according to Schmiedel's formula for calculating the **R**-value), the disks tended towards a horizontal position, independent of the position at start. At still higher **R**-values, 80 up to 300 (according to Schmiedel's formula for calculating the **R**-value), periodic oscillations set in, and the settling apparently took place in a zig-zag path of the type illustrated by Wetherell [50] by photographic reproduction of the settling of a small, flat crystal of silver nitrate in water acidulated with HCl.

Gans has given the question of settling position of disks and rod-shaped particles a mathematical treatment which may be summarized up in his own words: [51]

"Auf den bewegten Körper wirkt kein Drehmoment infolge der Druckkräfte der Flüssigkeit. Scheiben oder Stäbchen, die drei aufeinander senkrechte Symmetrieebenen besitzen, haben also nicht die Tendenz, beim langsamen Fallen in einer Flüssigkeit sich irgendwie einzustellen."

Thus flat particles have no tendency of taking any preferred settling position when their coefficient of resistance as a function of Reynolds number falls beyond and below the upper limit for the validity of Stokes's Law, i.e. at about Reynolds number 0.50. This does not mean, however, that the particle form ceases at this value to be a factor in the

[48] Otogorô Miyagi, "The Motion of an Air Bubble Rising in Water," *Phil. Mag.*, 6th Ser., **50**, 112–140, 1925.

[49] J. Schmiedel, "Experimentelle Untersuchungen über die Fallbewegung von Kugeln und Scheiben in reibenden Flüssigkeiten," *Physikalische Zeitschrift*, Bd. XXIX, 1928, pp. 593–610.

[50] E. W. Wetherell, "The Track of a Flat Solid Falling through Water," *Nature*, **CX**, 845, 1922.

[51] R. Gans, "Wie fallen Stäbe und Scheiben in einer reibenden Flüssigkeit?," *Sitzungsberichte der mathematisch-physikal. Klasse der K. B. Akademie der Wissenschaft zu München*, Bd. XLI, 1911, p. 197.

sorting of particles of different shapes, although the sorting is likely to be less distinct than for higher Reynolds numbers.

It was pointed out on p. 17 that the coefficient of resistance as a function of Reynolds number varies with the position of the particle relative to the fluid motion. A disk, settling with a "fair velocity" in an "incompressible" fluid, with its broad side at right angle to the direction of translation, exhibits a resistance which is mainly due to the dynamic pressure in the front and the "suction" in the rear. It is evident from the quotations above that this type of resistance becomes of increasing importance with increasing Reynolds number, because of the observed tendency of the particle to take a position with its broad side at right angle to the direction of translation. If, on the other hand, the disk moves edge-wise through the fluid, the head pressure will be much less and the generation of the vortex system in the rear will be insignificant or entirely absent. In this case the resistance is mainly due to the viscosity of the fluid and the surface area of the disk. We remember from our discussion on viscosity that the layer of fluid in immediate contact with the surface, i.e. the "boundary layer," adheres to the surface and is therefore at rest, and consequently retarding the velocity of the superimposed layers of fluid. In ordinary cases it is generally assumed that there is no motion relative to the solid, of the fluid immediately in contact with it. The contrary supposition would imply a greater resistance to the sliding of one portion of the fluid past another than to the sliding of the fluid over the solid.

P. Schulz [52] demonstrated that the "constant settling velocity" of a cube varied somewhat with its position. His data show that in general the slowest velocity is attained when the cube falls with its edge downwards. With one of its flat faces ahead the velocity of the cube increased somewhat. The greatest velocity was obtained with one of the *solid corners* [53] ahead, i.e. with the space diagonal of the cube parallel to the direction of translation. Although the variation of Schulz's data does not permit any generalization, there is no

[52] Op. cit.

[53] A *solid corner* is understood to be the intersection of three or more converging surfaces of a solid. The angular value may vary in the different cross-sections of the solid corner.

doubt that more delicate experiments will prove that there is a difference in the constant settling velocity, not only with the different positions of the cube, but that also the viscosity of the fluid may be a factor influencing the position. It has been previously pointed out that cubes of small volume and weight readily took a position with a flat face downwards, an observation which may indicate the tendency with decreasing Reynolds number.

If the settling velocity varies greatly with different positions of the solid, it seems that any sorting of granular material on the basis of shape would be impossible, except when the settling velocities of the particles are sufficiently high to influence the settling position, with reference to the largest cross-sectional area. Before discussing this question further, let us see what can be learned from experimental results with individual particles.

THE SETTLING VELOCITY AND THE DEGREE OF SPHERICITY OF A PARTICLE

The three principal properties of a solid are its specific gravity, volume and surface area. If we agree that deformation of a solid involves neither subtraction nor addition to the volume, then the principal factor in form alteration is the change of surface area. It is true that the geometric shape of a solid may be altered without changing either the surface area or the volume,[54] but any radical deformation must involve a change of the surface area. This subject has been more fully discussed in two previous papers.[55]

A classification of regular and irregular solids according to their similarity with standard geometric forms would involve personal opinion and would not permit any gradation from one type to the other. In the articles mentioned above the shape of irregular rock particles was expressed by the nu-

[54] This type of deformation, involving no extension or contraction of any part of the surface, may be properly called *bending;* see J. C. Maxwell, "On the Transformation of Surfaces by Bending," *Trans. Cambridge Phil. Soc.*, **IX**, Part IV, 445–470, 1856.

[55] Hakon Wadell, "Volume, Shape and Roundness of Rock Particles," *Journal of Geology*, **XL**, 443–451, 1932.

Hakon Wadell, "Sphericity and Roundness of Rock Particles," *Journal of Geology*, **XLI**, 310–331, 1933.

merical value of the *degree of true sphericity*. This conception is based upon the isoperimetric property of a sphere and expressed by the formula:

$$\frac{s}{S} = \psi,$$

where s is the surface area of a sphere of the same volume as the particle, S the actual surface area of the particle, and the Greek letter ψ is the symbol for the degree of true sphericity. The maximum value obtained by this formula is 1.000, which is the numerical expression for the degree of true sphericity of a sphere.

The separation of particles according to their shape has long been an important problem in connection with ore dressing, crushing and pulverization, in sand and gravel industries, in ceramics, and for the understanding of sedimentological processes in general. About 80 years ago V. Pernolet [56] experimented with settling velocities of rock particles of different forms. Because of the difficulty of comparing the irregular shapes of such particles with standard geometric forms, he wanted to find out more precisely how the geometric shapes influenced the settling velocity. He took to that end a number of small pieces of lead of about equal weight and formed them into various geometric shapes such as spheres, cubes, disks, plates, cylinders and prisms of varying length and thickness. These particles were dropped in a 6 meters long vessel filled with water. A record was taken of the time which each particle required to drop this distance.

I have recalculated Pernolet's geometric forms into terms of their degree of true sphericity. Figure *2a* illustrates the influence of the degree of sphericity on the settling velocity. Figure *2b* shows the coefficient of resistance as a function of the sphericity value.

It was shown in Table I that the "largest" cross-sectional area increases with an increase of the surface area, in other words, it increases with decreasing numerical value of the degree of true sphericity. In addition to the size of the

[56] V. Pernolet, "A l'étude des préparations mécaniques des minerais, ou expériences propres à établir la théorie des différent systèmes usités ou possibles," *Annales des Mines*, 4ME Serie, **XX**, 379–425, 535–596, 1851.

cross-sectional area, the size of the surface area of the solid is an important factor influencing the resistance, especially at low Reynolds numbers. In general, an increase of the surface area goes hand in hand with an increase of the largest cross-sectional area, both factors in the resistance which a solid exhibits in translation through a fluid. Finally it has also been shown that the degree of circularity of the cross-section, normal to the translation, influences the numerical value of the coefficient of resistance. A sphere has a maximum degree of sphericity, i.e. it equals 1.000, which also is the numerical value for the degree of circularity of any cross-sectional area through the center of the sphere. If a sphere is deformed, its largest cross-sectional area is always increased in size and frequently also deformed, i.e. its degree of circularity is changed. Considering an infinite number of cross-sections through the geometric center of a solid, we may say, as a generalization, that the degree of circularity decreases with decreasing degree of sphericity. I am, of course, aware that a sphere may be deformed to a disk, the "largest" cross-sectional area of which has the same degree of circularity as any cross-section of the sphere. This maximum degree of circularity for one single plane of the disk has, however, been attained by a considerable decrease of the circularity in all the other cross-sections of the solid, when it was deformed from a sphere to a disk. In conclusion, a decrease in the degree of sphericity involves (1) an increase of the surface area, (2) an increase of the largest cross-sectional area and also an increase of the sum of the areas of an infinite number of cross-sections, and (3) a general decrease in the degree of circularity of the planes through the center of the solid. Each of these changes is not necessarily absolutely proportional to the change in the degree of the sphericity, but taking the problem as a whole, it gives a theoretical foundation for assuming that the degree of sphericity is a factor influencing the settling velocity of a solid. This is in practice demonstrated in Fig. 2 *a* and *b*.

THE COEFFICIENT OF RESISTANCE AS A FUNCTION OF REYNOLDS NUMBER FOR SOLIDS OF DIFFERENT SPHERICITY VALUES.

It is evident that, when dealing with non-spherical forms, the size of the cross-sectional area, normal to the direction of

translation, varies with the position of the solid. Now that some uncertainty exists about the preferred settling position under different conditions of fluid motion, and since we frequently must count on disturbances of the fluid which, at least temporarily, influence both the settling position and the settling velocity of the solid, a calculation of the coefficient of resistance on the basis of any particular cross-sectional area of the particle would somewhat obscure our experimental data.

Attention is again called to the formula for the coefficient of resistance (equation no. 9, p. 17). If the solid is a sphere, the cross-sectional area A equals πr^2, and C_R represents the coefficient of resistance for a sphere. If the solid is non-spherical, then the numerical value of the coefficient of resistance reflects a difference in the conditions of the fluid motion and the resistance due to the deviation in shape from that of a sphere. It has been shown that, when the solid is deformed, the size of the largest cross-sectional area undergoes a change. The numerical value of the size of the area A in the equation no. 9 should then be adjusted for each particular form and position, because $v^2\rho_F/2$ represents the dynamic pressure per unit area of A, and $Av^2\rho_F/2$ expresses the total pressure on an area A normal to the direction of translation. It should be noted, however, that an adjustment of the numerical value of A for each particular form, not only is difficult in view of the discussion in the preceding paragraph, but we have thereby to some extent already made a correction for the shape with reference to its largest cross-sectional area. The result of such an adjustment would be that the numerical value of the coefficient of resistance C_R reflects the influence of other features of the shape rather than that of the entire shape, including the variation in size and plane shape of the largest cross-sectional area. If we therefore wish to obtain a numerical value of the coefficient of resistance which indicates the influence of shape alterations with reference to a sphere as a standard form of comparison, we insert in formula no. 9 a cross-sectional area πr_n^2 i.e., a cross-sectional area of a sphere of the same volume as the particle. It is evident that if the solid is a sphere then $\pi r^2 = \pi r_n^2$, where r_n is the true nominal radius, i.e. the radius of a sphere of the same volume as the particle. For the calculations of the coefficient of resistance

as a function of Reynolds number the following formulæ are introduced:

$$(10)\quad \text{Coefficient of resistance} = C_R = \frac{2V(\rho_S - \rho_F)g}{\pi r_n^2 v^2 \rho_F} = \frac{2 \cdot \frac{4}{3}\pi r_n^3(\rho_S - \rho_F)g}{\pi r_n^2 v^2 \rho_F} = \frac{\frac{8}{3} r_n(\rho_S - \rho_F)g}{v^2 \rho_F}$$

$$(11)\quad \text{Reynolds number} = \mathbf{R} = \frac{v d_n \rho_F}{\mu} = \frac{v d_n}{\nu},$$

where r_n is the true nominal radius, d_n the true nominal diameter, C_R the coefficient of resistance, v the translation velocity of the solid, ρ_S the specific gravity of the solid, ρ_F the specific gravity of the fluid, $g = 981$ (cm. sec.$^{-2}$), i.e. the acceleration of gravity, $\frac{4}{3}\pi r_n^3 = V$ = the volume of the solid, **R** Reynolds number, μ the viscosity coefficient of the fluid, and ν is the kinematic viscosity of the fluid.

The formulæ given above have been used for calculations of the coefficient of resistance as a function of Reynolds number on the basis of experimental data obtained from Schmiedel,[57] Pernolet,[58] Schulz,[59] and Richards.[60]

Schmiedel's data refer to the constant settling velocities of disks (Scheiben) normal to the direction of translation. The diameters of the circular areas of the disks range from 0.1948 cm. to 1.998 cm. Their thicknesses vary from 0.00489 cm. to 0.02067 cm. The disks consisted of aluminum, steel, silver and gold with a specific gravity ranging from 2.7 to 19.34. The fluids were composed of glycerin and homogeneous mix-

[57] J. Schmiedel, "Experimentelle Untersuchungen über die Fallbewegung von Kugeln und Scheiben in reibenden Flüssigkeiten," *Physikalische Zeitschrift*, Bd. XXIX, 1928, p. 605.

[58] V. Pernolet, "A l'étude des préparations mécaniques des minerais, ou expériences propres à établir la théorie des différent systèmes usités ou possibles," *Annales des Mines*, 4ME Serie, **XX**, 390, 1851.

[59] P. Schulz, "Neue Bestimmungen der Konstanten der Fallgesetze in der nassen Aufbereitung mit Hilfe der Kinematographie und Betrachtungen über das Gleichfälligkeitsgesetz," *Doktor-Ingenieurs Dissertation, v. k. Sächs. Technischen Hochschule zu Dresden in Verbindung mit der k. Sächs. Bergakademie zu Freiberg, 1914.*

[60] Robert H. Richards, "Velocity of Galena and Quartz Falling in Water," *Trans. Am. Institute of Mining Engineers*, **XXXVIII**, 219, 1907–08.

tures of glycerin and water with a kinematic viscosity ranging from 0.00995 to 1.391. The calculated sphericity values (ψ) of the disks range from 0.097 to 0.253, the majority of them falling within a close range of 0.125 on the one hand, and 0.220 on the other. In order not to give a confusing impression of the chart Fig. 3, only the values of those disks having a sphericity of 0.121–0.129 (difference 0.008) and 0.216–0.225 (difference 0.009) have been plotted. Taking into consideration experimental errors, the others fall according to their sphericity values, as a rule, within a proper range of the constructed curves.

With reference to my calculations of Pernolet's data the following should be noted. The heading of the table on p. 390 of his article reads: "Tableau n° II relatif à des corps de même densité et de même volume, mais de formes différentes tombant dans une profondeur d'eau de 6 metres." A recalculation of his data shows, however, that the lead particles vary somewhat in volume, and one solid, "Prisme IX," proved to have about four times less volume than the average of the rest. This is apparently due to an error in the text, an assumption confirmed by the calculated coefficient of resistance as a function of Reynolds number. This solid has therefore not been plotted in Fig. 3. The dimensions of some of his other solids vary somewhat for equal form types. For instance, his cylinders of type "III" are reported to have diameters ranging from 1.1 to 1.2 cm., and a variation in length (or thickness, "éppaisseur," as he called it) from 1.3 to 1.4 cm. There is, of course, in these cases a slight variation in the sphericity value, depending on which numerical value one cares to choose. In such cases the arithmetic mean of the sphericity values has been taken as an average for the given form type. The settling times of his spheres vary between 3 and 4 seconds, the higher value being more nearly correct as shown by the calculated value of the coefficient of resistance as a function of Reynolds number compared with Wieselsberger's results (Op. cit.). Finally, Pernolet gave only the total time which a particle required for a settling translation of 6 meters. Thus acceleration in the beginning, before the particle assumed a constant settling velocity, introduces a slight error (distributed over a fall distance 6 meters) in my values. The kinematic

viscosity has been calculated on the assumption that Pernolet used fresh water of 18° C.

Schulz determined the constant settling velocities of small pieces of marble, sphalerite, pyrite, quartz and coal in paraffine oil and water at temperatures of 18° to 19° C. He classified the shapes as spheres, cubes, "rundliche," "längliche," and "platte." In view of the vagueness involved in the last three terms, and since no data were furnished which permitted an exact calculation of the sphericity values for solids of these shapes, it was deemed unnecessary to include them in this study. The coefficient of resistance as a function of Reynolds number has been calculated for his cubes (artificial and natural) of marble, sphalerite and pyrite, and plotted accordingly in chart Fig. 3.

The data given by Richards refer to settling velocities of small pieces of galena of cubic shape. The experiments were undertaken by A. S. Warren and M. L. Nagel. Their records give the total time which a solid requires to fall a distance of 2.8 meters in water. Thus the acceleration in the beginning, before the particle assumed a constant settling velocity, introduces a slight error in my calculations. The kinematic viscosity has been calculated on the assumption that fresh water of 18° C. was used.

A study of the chart Fig. 3 shows that the coefficient of resistance as a function of Reynolds number reveals a rather sensible differentiation between solids of different sphericity values, even for low Reynolds number (Schmiedel's disks). The data for the cubes of Schulz, Pernolet and Richards coincide well, taking into consideration a certain amount of experimental errors. It is also to be noted that Pernolet's lead solids of approximately equal volumes but different sphericity values fall almost in a straight line from the sphere $\psi = 1.000$ to the disk $\psi = 0.237$. The general trend of the curve between Reynolds numbers 10 to 50 for Schmiedel's disks of a ψ-value 0.220 permits a tentative connection with Pernolet's disk of a ψ-value 0.237.

The fact that the curves become asymptotic towards lower Reynolds numbers is important, because it eventually permits a differentiation between aqueous and æolian deposits by a shape study of sedimentary particles. Since the viscosity

and density of air are distinctly different from those of water, sedimentary particles of a given small volume and weight are better sorted according to their shape in air than in water. I intend to return to this question in a forthcoming paper. For the present the conclusion is tentative in expectation of further results of a sufficient number of analyses.[61]

OTHER FACTORS INFLUENCING THE SETTLING VELOCITY AND THE RESISTANCE WHICH A ROCK PARTICLE EXHIBITS IN TRANSLATION THROUGH A FLUID.

It has been shown that the surface area, and the size and the degree of circularity of the cross-sectional areas influence the numerical value of the coefficient of resistance as a function of Reynolds number. These factors are, in all probability, the most important shape features influencing the fluid motion about the solid. A sphere has (1) the greatest relative volume with the smallest surface area, (2) the smallest cross-sectional area taken as an average of a great number of such sections, and (3) a maximum degree of circularity in all cross-sectional planes. Taking the average of the translation velocities of a great number of positions for particles of different shapes, the sphere has a greater settling velocity than any other solid of the same volume and density.

Other factors influencing the settling velocity are the roughness of the surface and the roundness of the corners and edges. The surface roughness can, at least for the present, be neglected, because its influence is comparatively small, and there is to my knowledge no indisputable, accurate method for its determination.[62] The question arises, what constitutes the surface roughness and which may be considered as part of the measurable surface? The roundness [63] of the corners and edges of a solid is recognized as a rather important factor influencing the resistance, especially at velocities above the

[61] A new analytical, sedimentographic method was announced in my preliminary paper, "Volume, Shape and Roundness of Rock Particles," *Journal of Geology*, **XL**, 450–51, 1932. The details and processes involved in this method will be given in a forthcoming article.

[62] In hydraulics distinction is made between the *absolute roughness* h, and the *relative roughness* h/r, where r represents the radius of the pipe and h the height of the roughness as measured from its base at the inner wall of the pipe.

[63] The reader is referred to definition and comments given in my earlier papers in *Journal of Geology*, **XL**, 446–49, 1932, and **XLI**, 318–23, 1933.

critical value. Vortices are more readily formed in the presence of sharp corners and edges than in the case of well-rounded ones. It should be noted that when the corners and edges are rounded off, the shape undergoes a certain change and the sphericity value is increased, but this increase varies for different shapes and is not in proportion to the increase of the roundness value. The degree of roundness as a factor influencing the numerical value of the coefficient of resistance is a problem best solved in aerodynamic laboratories.

Jordan [64] pointed out, and probably others before him, that, due to the position of the center of gravity, a pear-shaped solid settled with its longest axis in a vertical position. It is also well-known that the fluid motion in the case of solids of "stream line" or "easy shape" even at high velocities is irrotational, except in a layer of fluid in the neighborhood of the surface of the moving solid. It seems that such conditions would to some extent invalidate our assumption that the degree of true sphericity is a determinative factor in sorting of sedimentary particles. It should be noted, however, that in the sedimentation processes of nature, the air and water are generally in turbulent motion which constantly changes the settling position of the particles. It must also be kept in mind that the size of the largest cross-sectional area, for instance of a disk, influences the settling velocity in practically all positions, except when the disk moves edge-wise through the fluid, because the size of the projection area of the disk, inclined at a given angle with reference to a horizontal plane, is determined by the size of the largest cross-sectional area and the angle of inclination.

Odén [65] has pointed out that the *equivalent radius* differs with the settling position, but that the settling velocity of a particle may be taken as the average of the velocities in the different positions. He also observed a rotating motion for small biotite particles in sedimentation, and expressed the opinion that even for very small particles the settling position

[64] Op. cit., p. 200.

[65] Sven Odén, "Eine neue Methode zur mechanischen Bodenanalyse," *Internationale Mitteilungen für Bodenkunde*, Bd. V, 1915, pp. 257–311.

According to Odén the *equivalent radius* is the radius of a sphere having the same settling velocity as the particle of the same material.

is never constant, because the impact of the molecules sets the particle in a rotating motion, whereby the resistance is always changing. It is also to be noted that the sorting in nature is generally accomplished under the twofold influence of gravity and a horizontal or nearly horizontal current (the Spitzkasten-method of ore dressing) permitting a sorting according to the size of both the vertical and the horizontal cross-sections of the solid in a given position, thereby eliminating to a large extent the influence of any preferred settling position. In conclusion, since the sphericity value reflects (1) the size of the surface area, (2) the sum of the cross-sectional areas in a great number of positions, as well as the sum of the projection areas, and (3) the degree of circularity of the cross-sections, the average settling velocity of a particle is to a large extent determined by the degree of true sphericity.

Controlled laboratory experiments, bearing on the influence of the individual properties of particles on the process of sedimentation, is helpful for the interpretation of the results of sedimentographic analysis and for the construction of industrial processes. It is believed that a complete, standard chart of the type given in Fig. 3 would be useful for determining the sphericity value of rock particles with reasonable accuracy. Further experimental studies are planned to that end.

A MEMBRANE ANALOGY SUPPLEMENTING PHOTO-ELASTICITY.

BY

J. G. McGIVERN and H. L. SUPPER.[1]

ABSTRACT.

In this paper is described a new method of finding the *sum* of the principal stresses in a two-dimensional model by means of ordinate measurements on a stretched rubber membrane as suggested by Den Hartog. The method is applied in detail on a bar with central circular hole in tension and is found to be easy in operation and of good accuracy.

I. INTRODUCTION.

While the analytical theory of elasticity has wide applications in the solution of engineering problems, it becomes very complicated and laborious when investigating bodies that have not the very simplest of geometrical shapes. It is because of this limitation of the mathematical theory that so much attention has been centered on photo-elasticity as a method for determining the *difference* of the two principal stresses at any point in a body of uniform thickness subjected to a system of loading in its own plane. Since this difference of the principal stresses is equal to twice the shear stress, this analysis becomes very useful when the object to be investigated is ductile and fails according to the maximum *shear* theory. Many of our machine parts, however, are hard and brittle and fail according to conditions predicted by the maximum *normal stress* theory. Therefore, paralleling the development of photo-elasticity there have been attempts to supplement its results by other methods, making it possible to obtain the *sum* of the principal stresses and hence from the two the individual principal stresses. The methods proposed have not been sufficiently accurate or simple in their execution to warrant their widespread adoption on the part of engineers.*

*(Added in proofs): Since the completion of this work an interesting new contribution was made to the subject by M. M. Frocht, See *Jour. Frank. Inst.*, July 1933.

[1] Graduate students at the Harvard Engineering School.

In this contribution, the authors apply a new procedure, employing a membrane,[2] that they believe is both simple and accurate. This paper will first consider those essentials of photo-elasticity necessary for a proper explanation, give a general description of the membrane analogy and its technique, and finally show its application to a typical problem.

II. ESSENTIALS OF PHOTO-ELASTICITY.

The principle of photo-elasticity was discovered by Sir David Brewster in 1816 when he found that when plane polarized light is passed through a transparent plate under stress, the polarized ray is broken up into two plane rays which vibrate in planes at right angles to each other and suffer retardation such that they are out of phase with each other by an amount proportional to the difference of the principal stresses. This phase difference causes interference effects so that if a stressed specimen be viewed with white light, a pattern of color bands results, or if a monochromatic light source is used, the result is a pattern of alternating black and white fringes. Since the optical law governing this behavior is dependent only on the elastic and optical constants of the material, its thickness and the value of the difference of the principal stresses (and not on the geometrical shape), it is possible to determine the stress difference value from either the color or the black fringe pattern. The calibration of the constants is done by taking a simple tension specimen and measuring the loads corresponding to the various color values, or also by using a simple beam subjected to a constant known bending moment. In the latter case the stress is calibrated for various points in the beam and then divided by the fringe order to give the stress difference value for each fringe. This value will of course vary with the material used and its thickness. For the tests described in this paper, the most sensitive material existing (phenolite[3]) was used with a

[2] J. P. Den Hartog, *Zt. f. angew. Math. & Mech.*, 1931, p. 156.

[3] Developed by Z. Tuzi; see "A New Material for the Study of Photo-Elasticity," *Scientific Papers of the Inst. of Phys. & Chem. Research*, Tokyo, Vol. 7, Oct. 1927; also *Proc. Third Intern. Congress for Appl. Mech.*, Stockholm, 1930, Vol. II, p. 176.

calibration of 264.3 lb./in.^{2} per fringe of green mercury light.

It is further possible by pure optical methods to obtain the directions of the principal stresses as well as the value of their difference. This was discovered by Clerk Maxwell when he conceived the idea of using two polarizers having their axes at right angles and located on different sides of the specimen. By keeping the axes at right angles, and revolving them together, it is possible to obtain a series of black lines called "isoclinics." Each one of these isoclinics has the property of being the locus of all points having the directions of their principal stresses the same as that of the setting of the axes of the polarizers. This follows since those rays which vibrate in the plane of the axes of the first polarizer cannot pass through the second polarizer (called analyzer) as they have no component parallel to its axis and therefore appear black.

Since the maximum shear stress is equal to one-half the difference of the principal stresses and acts on a plane 45° from the plane of the maximum stress, it is possible by pure optical means to determine both the magnitude and direction of the maximum shear stress.

As has been stated before, it is sometimes necessary for the engineer to know the value of the principal stresses P and Q individually. Knowing then $(P - Q)$ from photo-elasticity, the usual procedure is to find $(P + Q)$ by some different method and then to determine the individual stresses P and Q separately.

III. VARIOUS METHODS FOR OBTAINING P AND Q SEPARATELY.

Coker in 1914 proposed a procedure which was originally suggested by Mesnager and consists of determining the sum of the principal stresses $(P + Q)$ by measuring the lateral contraction. By means of this measurement, together with a knowledge of Poisson's ratio and Young's modulus, the value of $(P + Q)$ can be determined at any point. This method has the serious disadvantage that the deformations to be measured are so small as to render reliable results very improbable in practice.

A method of graphical and numerical integration based on

the equations of Mesnager was developed by Filon.[4] The calculations are started at a boundary where P and Q are known, and then continued along a stress trajectory (which is a line having the principal stress cross as tangent and normal at each point). The stress trajectories can be obtained graphically from the isoclinics, but the whole procedure is very long and tedious as well as quite inaccurate in certain regions of the object.

A third method originated by Favre[5] is a purely optical one, requires the use of an interferometer and consists of breaking up the original plane polarized ray into two parts: one passing through the stressed specimen and the other dodging around it. If the plane of the polarized light at a certain point of the specimen is parallel to its principal stress direction, then the retardation of that ray is proportional either to P or to Q. Thus the two principal stresses are determined independently by comparing their retardation with respect to the unaffected ray. It is, however, necessary that the directions of P and Q for each point be determined first, by the ordinary isoclinic method, before the angle of the polarizer can be set for finding the retardations of P or Q for a particular point. This method is not of great practical importance, as it requires very sensitive precision apparatus, a large amount of time and a very high degree of technical skill for its operation.

The membrane method, which is the subject of this paper, requires a moderate amount of rather simple apparatus, gives results with less work than any of the three procedures discussed above, and is also inherently more accurate.

IV. GENERAL DESCRIPTION OF MEMBRANE ANALOGY AND APPARATUS.

Due to the fact that at the unloaded boundaries of the specimen one of the principal stresses disappears, it is possible

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[4] L. Filon, "On the Graphical Determination of Stress from Photo-Elastic Observation," *Engineering*, Oct. 1923.

[5] H. Favre, "Sur une nouvelle méthode optique de détermination des tensions intérieures." Publications du Laboratoire de Photoélasticité de L'Ecole Polytechnique Fédéral de Zürich, 1929.

from a photo-elastic photograph to evaluate $(P + Q)$ along all free boundaries since $(P - Q)$ equals $(P + Q)$. Now it so happens that the equation of the small ordinates of a membrane stressed with a constant tension and having the same pressure on both sides is the same as the equation defining the distribution of $(P + Q)$. This common equation is known as the differential equation of Laplace and is written for the two cases as:

The equation for the ordinate z of the membrane [6]

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0. \tag{1}$$

The equation for the distribution of $(P + Q)$ [7]

$$\frac{\partial^2 (P + Q)}{\partial x^2} + \frac{\partial^2 (P + Q)}{\partial y^2} = 0. \tag{2}$$

In order to have a complete similarity it is only necessary to satisfy the same boundary conditions. Since the boundary values of $(P + Q)$ can be obtained from photo-elasticity, we have only to make the *boundary* ordinates of the membrane proportional to the $(P + Q)$ values to have complete similarity between the $(P + Q)$ values and the z values of the membrane in the *interior* of the specimen also.

Some care has to be exercised about evaluating the sign of $(P + Q)$ at the boundary. If the signs at the boundaries are not taken correctly, serious errors can be made. This point will be discussed in detail in the next section.

The requirement of a "membrane" as it is assumed in the derivation of equation (1) is that the tension in it is always constant, independent of any change in the shape. This condition would be fully satisfied by using a soap film where the tension is a function only of the capillary constant of the film. Although some investigators have succeeded in making soap films that have lasted for several hours, this entails great care in both its making and its use. For this reason we have employed a rubber membrane, which with moderate care (protecting it from direct sunlight) lasts for over a month. If we so select the scale of our boundary ordinates that the tangent of the largest slope of the membrane will be one-fifth,

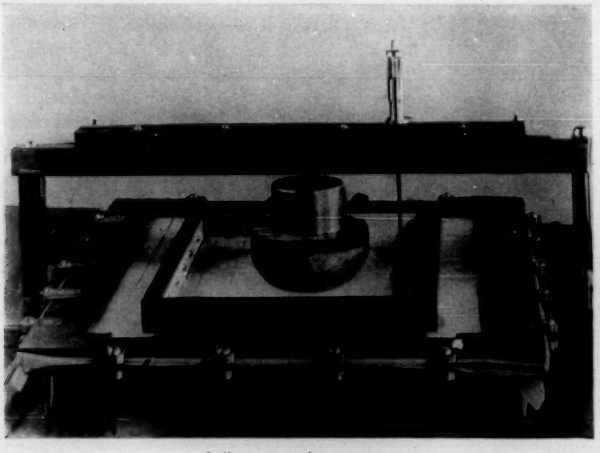
[6] Love, Mathematical Theory of Elasticity, 4th Ed. p. 322.

[7] Love, p. 206.

then the error introduced by a change in tension due to the change in shape is less than 2 per cent. In the derivation of equation 1 it is also assumed that the angle of slope is so small that its tangent can be taken as equal to its sine. The error due to this assumption occurs in the soap film as well, and is also of the order of 2 per cent.

The conditions that the rubber membrane be stressed with equal tension was fulfilled by ruling it into one-half inch squares and stretching it until the original small squares became one inch on a side. This was accomplished by fastening clamps, one inch wide, alternately around the edge of the rubber and fastening these clamps with strings to a large frame so that the membrane could be stretched until the original half inch space between the clamps became one inch. The center portion of the rubber was then clamped between two permanent frames especially made for that purpose. These frames are marked *A* in Fig. 1. The boundary ordinates of the rubber membrane, as prescribed by the photo-elastic test, are forced on it by a lower and an upper frame (*B* in Fig. 1) carrying celluloid inserts filed to the proper contour.

FIG. 1.



Ordinate measuring apparatus.

The ordinate measuring apparatus consists of a base plate on which slides an overhead beam which allows a drop micrometer to move along it. By means of scales on both the base and the beam, it is possible to definitely locate any point (x, y) on the model and measure its height z. Figure 1 shows the complete apparatus set up and ready for use.

V. APPLICATION TO A DEFINITE CASE.

The specimen selected for applying the new technique consists of a simple phenolite tension member with a circular hole, equal to one-half of its width, centrally located in it. The fringe photograph for this model is shown in Fig. 2.

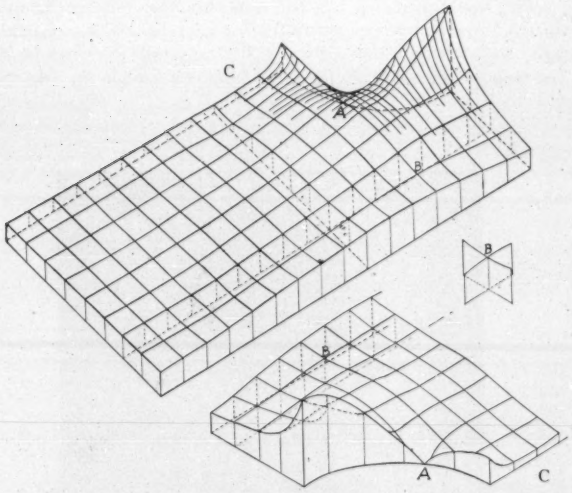
FIG. 2.

C

Fringe photograph of $(P-Q)$-pattern.

On account of spherical aberration, it is of importance to rule a network of squares on the original specimen and photograph it together with the $(P - Q)$-pattern. Figure 3 represents the $(P - Q)$ stress surface for one-quarter of the model. The photograph of Fig. 2 can be considered to be a civil engineer's contour map of the surface of Fig. 3.

FIG. 3.



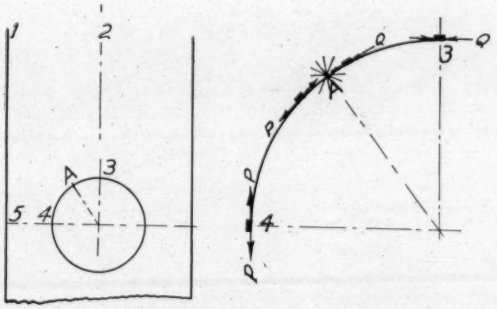
$(P - Q)$ stress surface.

Since the P stress was taken as the longitudinal stress at a distance far removed from the hole, it is seen that the maximum P stress is tensile and is equal to approximately three times the average applied stress. This maximum is located at the surface of the hole on the horizontal center line (point 4 in Fig. 4). The maximum Q stress is approximately equal to the average applied stress, is compressive, and located at point 3 in Fig. 4. We know it to be compressive because in Fig. 3 $(P - Q)$ is positive and P is equal to zero. An

investigation of P and Q around the hole presents some interesting features, as can be explained by referring to Fig. 4.

Along the lines 1–5, 5–4 and 2–3 in Fig. 4, the direction of the P stress is vertical and has a positive value, with the exception that at the point 3 it is zero. The Q stress remains horizontal in direction along these same lines and is equal to zero at the points 1, 5 and 4. Considering the contour of the hole, it is seen that at point 4 the P stress has a definite value and the Q stress equals zero, while for point 3 the reverse

FIG. 4.



Direction properties of inner boundary.

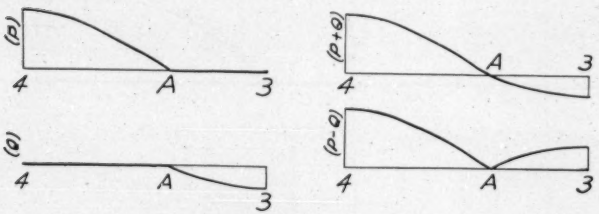
is true. Along the circular contour of the hole the stress cross is tangential and normal, so that in moving from point 3 to point 4 that cross turns through 90 degrees, and if nothing particular happens, the P stress, being vertical at 4, would become nearly horizontal near point 3, which leads to an inconsistency.

In order to avoid this, it is necessary to reverse the meaning of P and Q at some point between 3 and 4. This naturally can happen only at a point where both these stresses become zero. This very important point is denoted by the letter A in Fig. 4, while Fig. 5 illustrates the relation still more plainly. The directional properties of this singular point of zero stress A located approximately at 36 degrees from the vertical centerline will be best understood when we

refer to the isoclinic and stress trajectory diagrams shown in Figs. 9 and 10.

Now we can proceed to the finding of the $(P + Q)$ values along the boundaries from the known values of $(P - Q)$ in Fig. 3. Along the outside boundaries $Q = 0$ and therefore $(P + Q) = (P - Q)$ without further difficulty. The same is true at point 4 of the circular inner boundary or, for that matter, at any point between 4 and A. However, at point 3 we have a different situation. Here $P = 0$ and Fig. 3 shows that $(P - Q)$ is positive so that Q must be negative. Consequently $(P + Q)$ is also negative and equal in numerical

FIG. 5.



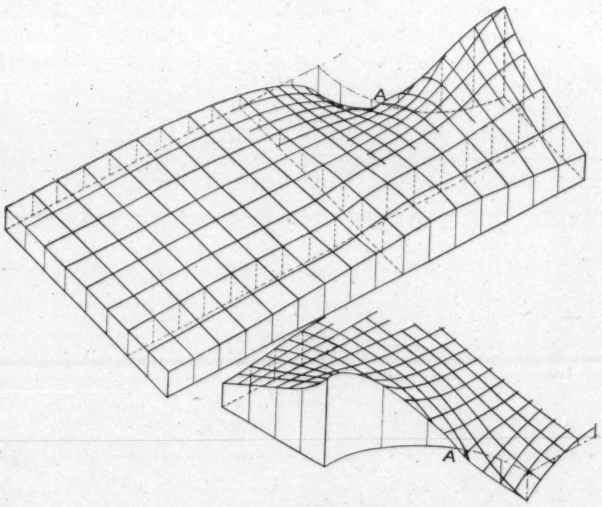
Stress distribution along inner boundary.

value to $(P - Q)$. This reversal of sign between $(P + Q)$ and $(P - Q)$ holds for the whole boundary between points 3 and A. This is further illustrated in Fig. 5. The value and the sign of the boundaries having been settled on, it is an easy matter to lay them out, attach them to their respective frames and by means of the set-up shown in Fig. 1 force them on the rubber membrane. The ordinates measured on the membrane surface thus formed are shown in Fig. 6.

The resulting $(P + Q)$ surface of Fig. 6 on first sight does not appear very different from the $(P - Q)$ surface of Fig. 3 with the exception of the change in sign at the boundary from A to 3. There are, however, two regions, lettered B and C on Fig. 3, which do not appear in Fig. 6. Point B indicates a "saddle point" in the $(P - Q)$ surface. If we proceed in a longitudinal direction through this point, we go through a maximum $(P - Q)$ value, while if we travel transversely

through it a minimum $(P - Q)$ value is passed. This point B is of importance in interpreting the fringe order and is easily recognized on the fringe photograph of Fig. 2. Point C indicates a bowl in the surface and a minimum value of $(P - Q)$ is obtained irrespective of the direction one passes through it. This point is also easily recognized in Fig. 2.

FIG. 6.

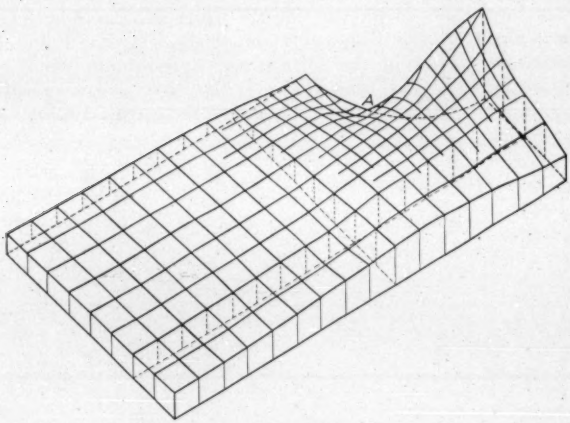


$(P + Q)$ Stress surface.

It is interesting to note that it would be impossible for any $(P + Q)$ surface to contain a bowl or a hill top since its differential equation requires that the sum of the two principal curvatures through a point must always be equal to zero. It is this characteristic that makes it possible to distinguish between a $(P + Q)$ and a $(P - Q)$ stress surface at first sight.

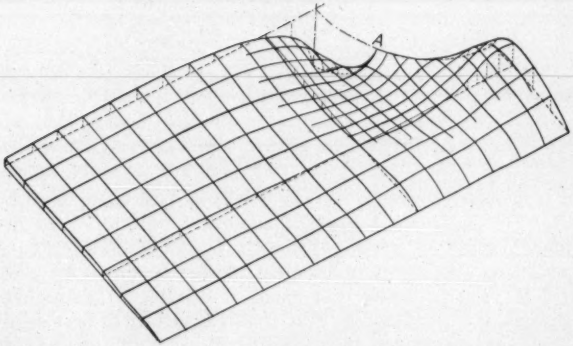
Having catalogued the values of $(P - Q)$ and $(P + Q)$ it is an easy matter to solve for P and Q separately. The P stress surface and the Q stress surface are represented in the

FIG. 7.



P – Stress surface.

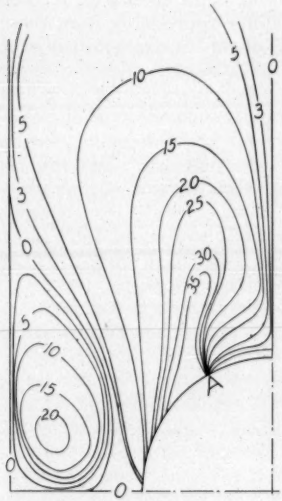
FIG. 8.



Q – Stress surface.

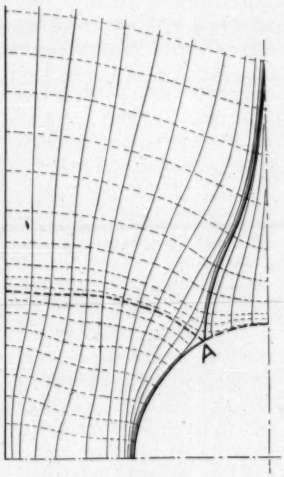
Figs. 7 and 8 respectively. The P stress surface does not vary much from either the $(P - Q)$ or the $(P + Q)$ surfaces with the exception of the region from point A to point 3 near the edge of the hole. This similarity is due to the small values of Q except for the region indicated. Since the Q stress is so small, its ordinate scale in Fig. 8 was taken double that of Fig. 7. The P stress diagram makes it possible to apply a static check on the analysis, since the area under the curve at the center section between 4 and 5 in Fig. 4 should equal the area under the curve at a large distance from the hole (between 1 and 2 in Fig. 4).

FIG. 9.



Isoclinic lines.

FIG. 10.



Stress trajectories.

Figure 9 shows the isoclinic lines which were determined experimentally by the usual method. These lines show the significance of the singular point A by the fact that all the isoclinics pass through it. From these isoclinics the stress trajectories shown in Fig. 10 were drawn. There are two

"singular trajectories," one represented by a heavy full line which shows the change in the direction of the P stress at A and the other represented by a heavy dotted line representing the change in the direction of the Q stress at A.

The behavior in the immediate neighborhood of A naturally is not affected by the width of the plate. Since an analytical solution is available for an infinitely wide plate,[8] it is possible to solve for the angle that the principal stress makes with the radius. The tangent of twice this angle was reduced to a linear function of both the angular displacement from the singular point and the ratio of the radii of the point in question to the radius of the hole. This made it possible to show the detail in the region of point A in Fig. 10. The trajectories as given in Fig. 10 differ appreciably from those published by Coker on page 968 of the December 1920 issue of the *General Electric Review*.

[8] Coker & Filon, "Photoelasticity," p. 484.

SPECTROPHOTOMETRIC DETERMINATION OF TONE INTENSIFICATION IN COLOR MOTION PICTURE FILM.

BY

ALAN M. GUNDELFINGER, B.S. in Ch.E.

ABSTRACT.

A spectrophotometric method of determining total visual transmission and visual density of non-neutral light filters is described. In addition, this method is utilized in determining density and gamma intensification produced by tones in color motion picture work and the importance of intensification control is emphasized.

BLACK AND WHITE PROCESS.

In the processing of black and white motion picture prints, certain controls, recognized by most of the laboratories of the motion picture industry as essential and valuable for the maintenance of economical and uniform production, are quite generally understood by the technologists and technicians of the industry. The more outstanding of these controls, as typifying the modern laboratory, may be enumerated as follows:

(*a*) Photographic or photometric tests of cinex and printer lights at regular intervals which serve the purpose of preventing general density fluctuation of both picture and sound track and also prevent distortion due to variation in color temperature or printer coefficient.

(*b*) Sensitometric control with reference to gamma. The maintenance of a constant gamma is the assurance of constant contrast in picture and the maintenance of the proper gamma is the inhibitor of distortion of sound.

(*c*) Sensitometric control with respect to inertia or the point of intersection of the H. & D. curve with the $\log_{10} E$ axis. Variation of this point may be the criterion of developer life, or, if the variation is known, may have its effect neutralized by a change of printer light range as is frequently done when a new emulsion is inducted into the laboratory. If inertia variation be ignored, then the picture will, of course, vary in general density and the sound will be materially affected with particular reference to amplification and distortion.

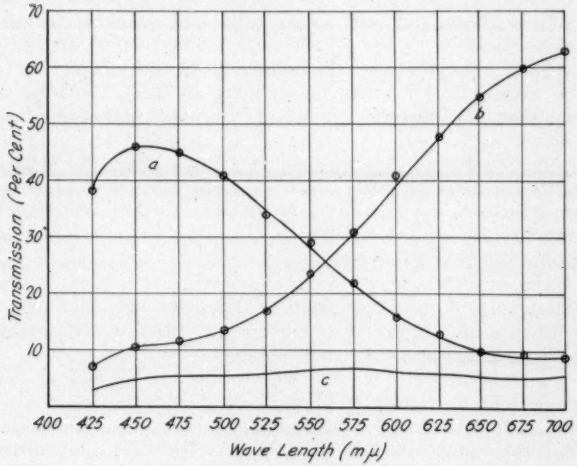
(*d*) Chemical analyses of hypo and hardening solutions and wash water. This insures proper fixing, hardening and washing.

(*e*) Dry bulb and wet bulb thermometric tests of dry box air to maintain constant drying conditions.

COLOR PROCESS.

In the processing of color motion pictures, it is essential, obviously, to maintain the aforementioned methods of control. Another factor, however, is introduced, which, if not closely observed and controlled, may nullify all the effort exerted in previous control and may cause the ruination of both picture and sound track. This factor is the intensification of both density and gamma as the result of toning.

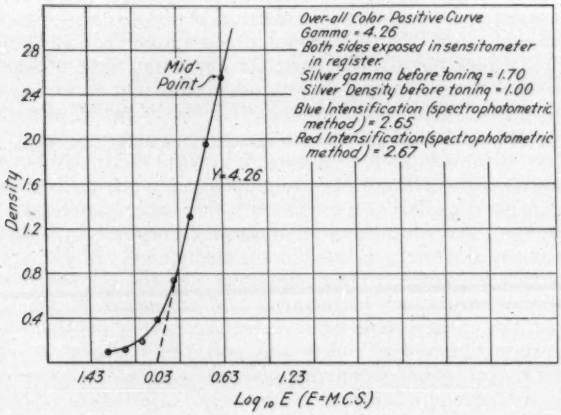
FIG. 1.



In order to illustrate the effect of tone intensification, it is necessary to refer to Figures Nos. 1 and 2. The former represents the spectral transmission curves of a typical iron tone and a typical uranium tone, (*a*) and (*b*) respectively.

The curves were obtained by plotting transmission versus wave length in millimicrons and the data for the curves were obtained from transmission observations at various narrow bands of wave lengths through toned silver densities. The original silver densities, as determined with a Bausch and Lomb polarizing densitometer, for curves (*a*) and (*b*) were 0.21 and 0.21 respectively. The film stock used was Eastman Duplitized No. 1509 and the densities were produced by a cinex exposure with subsequent development in the regular

FIG. 2.



process developer under standard operating conditions. The developed densities were then determined, after which the strips were sent through the color machine under standard operating conditions. When fully processed, the strips were then placed, singly, in a Bausch and Lomb Universal Spectrophotometer wherein, and by means of which, the spectral transmission data were obtained. The average magnitude of the spectral bands under observation was of the order of 5mμ.

Curve (*c*) was obtained by plotting the products of the corresponding ordinates, of curves (*a*) and (*b*), versus wavelength and represents the spectral transmission of the blue and

red strips, superimposed. It may be readily observed that with the exception of a slight peak in the green region of the spectrum, transmitted light through the two toned strips superimposed would appear fairly neutral to the human eye, provided, of course, that the incident light were white. As a matter of fact, daylight when transmitted through the combined strips has a faint greenish hue.

SENSITOMETRIC DETERMINATION OF TONE INTENSIFICATION.

With this thought in mind, reference is now made to Figure No. 2. The curve, therein, represents an H. & D. curve of gamma *4.26*, produced by plotting density versus $\log_{10} E$ where E = M.C.S. The film stock used was Eastman Duplitized No. 1509–36. The data for the curve were obtained in the following manner. Both sides of the film were exposed in register in an Eastman Type II *b* sensitometer using the regular positive exposure set up (Color temp. = 2800° K.). The strip was then developed in the positive developing machine under exact operating conditions, attached to another strip similarly exposed on one side only. After drying, the black and white densities of the single exposure strip were read on a Western Electric densitometer and the H. & D. curve was plotted. The gamma was 0.85 and the mid-point density was 0.50. The gamma and mid-point density on each side of the double exposed strip were, therefore, assumed to approximate very closely 0.85 and 0.50, respectively, and the overall gamma and mid-point density of the strip to approximate 1.70 and 1.00, respectively. The double exposed strip was then colored in the coloring machine under precise operating conditions, one side being toned blue and the other side, red.

When dry, the strip presented to the eye a series of fairly good neutrals with a slightly greenish hue. It was possible, however, on account of the very slight amount of color, to obtain a fairly good intensity balance in the densitometer and therefore make possible a close approximation of the densities. The densities were, consequently, determined and plotted against $\log_{10} E$ where E = exposure in meter candle seconds of corresponding frame on each side of the strip.

Since

$$D = \gamma(\log_{10} E - K),$$

if γ is increased by some factor, D must also be increased by the same factor.

It may be observed from the curve that γ after toning was increased from 1.70 to 4.26 or by a factor of 2.51 and that the mid-point density was increased from 1.00 to 2.55 or by a factor of 2.55. The average gamma or density intensification factor for each of the tones, therefore, as determined by this method is 2.53.

This method of determining intensification factors of tones has been set forth only for the purpose of illustrating the results of intensification as pertains, particularly, to formation of neutrals in color pictures. Before the method is considered, it might be advantageous to consider, first, the effect, on color motion picture quality, of variation in intensification.

EFFECT OF INTENSIFICATION VARIATION ON QUALITY.

It is obvious from Figs. 1 and 2 that for the best reproduction of neutral objects, both silver gamma and color gamma or compensating values of each on each side of the film should produce equal gammas. Experience has proven that three changes may result insofar as the tones are concerned. They, with their effects, are listed as follows:

1. Intensification of blue tone may increase, that of the red may decrease, or both may occur simultaneously. In any case, the picture will be predominated by blue, and may vary somewhat in general density. The neutral objects, progressing from light to dark, will take on a blue hue. The sound track, if it is toned, will be materially affected, if the intensification of that tone varies.

2. Intensification of the red tone may increase, that of the blue may decrease, or both may occur simultaneously. In any case, the picture will be predominated by red, and may vary somewhat in general density. Likewise, if the sound track is toned and that particular tone varies, the quality of sound will be materially affected.

3. Intensifications of both the blue and red tones may increase or decrease simultaneously. If the former occurs, the picture will assume a more contrasty appearance and its general density will increase. If the latter occurs, the opposite effect will be noted. The quality of sound in either case, if the track is toned, will be affected.

For uniform production, therefore, and in the interest of good quality, it is obvious that intensification of the tones must not only be known, but also controlled.

WEAKNESS OF SENSITOMETRIC CONTROL OF INTENSIFICATION.

Returning to the sensitometric method of determining intensification factors, as outlined previously, there are certain salient disadvantages which make it impracticable or undesirable. The very fact that, in this particular case, and certainly in the majority of cases involving other tones, the combined blue and red densities give some sensation of color to the eye, eliminates any high degree of accuracy obtainable in the determination of density with a densitometer. Then, too, the method gives only an average intensification produced by each tone, for the red and blue gammas can only be determined together. Finally, in the case of widely separated intensifications or color gammas, the hue increases with increasing density, thereby making an intensity balance in the densitometer a more difficult feat and, as a result, reducing still more the accuracy of the method. In the latter instance, also, the determination of an average intensification becomes more meaningless. The method, therefore, is not one to be desired or recommended.

CHEMICAL CONTROL.

There is no question or doubt concerning the importance of chemical analysis and control as an aid in maintaining consistent and proper intensifications of tones, as variation in the composition of the toning solutions themselves is undoubtedly the greatest contributing factor to intensification variation.

A typical blue toning solution contains the following constituents:

Ferric Ammonium Oxalate,
Potassium Ferricyanide,
Ammonium Chloride,
Hydrochloric Acid.

It is a matter of common knowledge that the toning reaction involves mainly the silver in the film and the ferric and ferricyanide ions. Analyses of the total quantities of those

ions including pH value are accurate, simple, and quick. Determinations of the potassium, chloride, ammonium, and oxalate ions, while perfectly possible, are not quite so simple and involve, in some cases, considerable care and time. Now while these ions do not actually enter into the toning reaction, their concentrations, singly and combined, materially affect the concentrations of the available reacting ferric and ferricyanide ions on account of the law of mass action, and probably affect side reactions. In addition, it is somewhat doubtful as to whether or not the knowledge of the exact concentrations of these ions could be utilized in interpreting the results which might be expected without some highly involved considerations.

In addition, if the analyses of all the ions were easily feasible, it would be impossible to maintain the concentration of all the ions constant with any system of replenishment owing to the differential removal of the various ions by the toning and static reactions encountered.

Finally, the intensification, of tones, is not a function only of the composition of the solutions, but is also dependent upon the condition of the gelatin, the type of grain produced by development, the extent of residual chemicals in the gelatin from previous operations and undoubtedly is somewhat affected by atmospheric conditions in the drying operation.

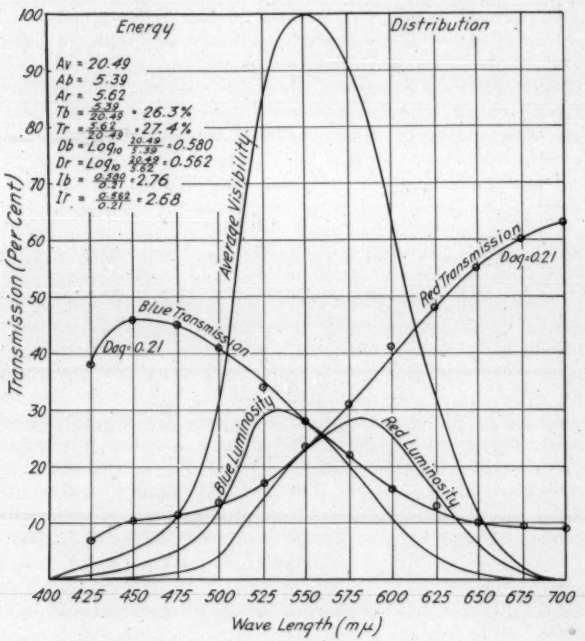
As a matter of efficient and utilitarian control, weight must be given to the feasibility, the element of time, and the labor involved. If the method is difficult, errors are likely to occur; if it requires considerable time, it is useless from an operating standpoint; and if it involves considerable labor, it becomes costly. Then, too, if the method does not include the analysis or control of *all* the contributing factors, it is certainly not the final answer.

The final answer or control, therefore, appears to be a photometric one which, at least, shows accurately what is happening in the toning process.

SPECTROPHOTOMETRIC DETERMINATION OF INTENSIFICATION.

In describing this method of determining tone intensification, frequent reference must be made to Fig. 3. It may be noted that the same blue tone and red tone transmission curves

FIG. 3.



are therein constructed as appear on Fig. 1. They are designated "Blue Transmission" and "Red Transmission" respectively and the silver densities before toning were for both 0.21. The data were obtained as previously described at the beginning of this paper.

In addition, it may be noted that an average visibility curve, so designated, has been constructed. The data for this curve were compiled by the *U. S. Bureau of Standards* [1] and represent the relative visibility of the human eye to each wave-length of the visible spectrum for light of constant energy distribution.

Inasmuch as this method, in the majority of cases, would be utilized for purposes of control, only, and the energy distribution of projector light sources is not only not known in practically all cases, but also is extremely variable, let it be assumed that the light source under consideration is one of constant energy distribution. As a matter of fact, noon daylight has practically constant energy distribution.[1]

If the ordinates of the average visibility curve multiplied by the corresponding ordinates of the energy distribution curve of the light source under consideration are plotted versus the corresponding wave-lengths of the spectrum, a curve would be obtained which would describe diagrammatically the relative intensities, of each wave-length of the light source, on the human eye. This curve would be designated as the luminosity curve. Inasmuch as the energy distribution curve has been assumed to be constant as shown on Fig. 3, the average visibility curve in this case becomes the luminosity curve.

If, then, the ordinates of the blue and red transmission curves are multipled by the corresponding ordinates of the average visibility (luminosity) curve, two curves are obtained, designated, respectively, as "Blue Luminosity" and "Red Luminosity" on Fig. 3. Each curve describes diagrammatically the relative intensities, on the human eye, of each wave-length of light, passing through the toned area, as compared with the intensity of the corresponding wave-length of the incident light.

If, now, the ordinates of the average visibility (luminosity) curve were to be added together, the summation would represent the total or integrated physiological effect of the light source on the human eye. Likewise, the summation of all the ordinates of the "Blue Luminosity" or "Red Luminosity" curve would represent the total physiological effect of the transmitted light on the human eye. By total or integrated physiological effect is meant the total intensity of light as seen by the eye, irrespective of color. The summation of the ordinates of each curve is represented by the area included between the curve and the "wave-length" axis. The areas can be determined easily and accurately with a planimeter.

By virtue of this reasoning, then, the total visual transmission of the blue and red toned areas may be determined by the following respective expressions:

$$T_b = \frac{A_b}{A_v}$$

and

$$T_r = \frac{A_r}{A_v},$$

where T_b = total visual transmission of blue toned area,
T_r = total visual transmission of red toned area,
A_b = area of "Blue Luminosity" curve,
A_r = area of "Red Luminosity" curve,
and A_v = area of Average Visibility (Luminosity) curve.
Since

$$D = \log_{10} \frac{\mathrm{I}}{T},$$

where D = density and T = transmission, then

$$D_b = \log_{10} \frac{A_v}{A_b}$$

and

$$D_r = \log_{10} \frac{A_v}{A_r},$$

where D_b = visual density of blue toned area
and D_r = visual density of red toned area.

Having determined the visual densities of the toned areas in this manner and having previously determined the silver densities before the toning operation, the intensification factors of each tone are calculated simply as

$$I_b = \frac{D_b}{D_{Ag}}$$

and

$$I_r = \frac{D_r}{D_{Ag}},$$

where I_b = intensification factor of blue tone,
I_r = intensification factor of red tone,
and D_{Ag} = silver density of toned area.

SUMMARY.

An example of this spectrophotometric determination of intensification is set forth on Fig. 3 using the same nomenclature as given above in the general expressions.

In addition it may be noted that, on Fig. 2, while the average intensification factor for each tone as determined by the sensitometric method is 2.53, the blue and red tone intensification factors, as determined by the spectrophotometric method, are 2.65 and 2.67 respectively. Taking into consideration the inherent faults of the sensitometric method as previously described and also the fact that the intensification factors were determined spectrophotometrically with the assumption of constant energy distribution of the light source whereas the energy distribution of the incandescent light in the densitometer is not constant, the factors, as determined by each method, appear to be in remarkably close agreement.

As for the practical importance of intensification control, the intensification factor of each tone has been observed to fall at various times, actually as low as 1.9 and to rise as high as 3.2. The significance of such variation must be evident when it is realized that the final densities and gammas, in the case of the two extremes, would be in the ratio of $\frac{3.2}{1.9}$ and in the event that one tone should produce a high factor at the same time that the other should produce a low factor, the utter impossibility of both color and neutral rendition must be evident.

If it is desired to determine the actual total visual transmission or visual density of a color filter by the spectrophotometric method, it must be borne in mind that the actual energy distribution of the light source under consideration must be utilized in constructing the luminosity curves.

The author wishes to express appreciation to Mr. Jerome W. Stafford for supplying the necessary data for Fig. 2.

REFERENCE.

1. GIBSON, K. S., AND TYNDALL, E. P. T.: "The Visibility of Radiant Energy," Bureau of Standards Scientific Paper (1923), No. 475, p. 174.

A Pound of Vitamin-C.—(*Chem. and Ind.*, Vol. 53, p. 113–23.) In a Jubilee Memorial Lecture on "Biochemistry and the Manufacture of Fine Chemicals," F. H. CARR tells a very interesting story of how so great a quantity (one pound) of Vitamin C suddenly came into existence. It was indeed an amazing accomplishment especially since the investigator, Szent-Györgyi had tried hard for several years to isolate this vitamin from the better known varieties of fruits and vegetables. In fact, his method applied to the extraction of 5000 oranges produced only a very minute quantity of the much sought for "ascarbic acid," the chemical name for Vitamin C. When one stops to realize that the daily human requirement of this anti-scorbutic (scurvy preventative) vitamin is not over one five-hundredth of an ounce and that as little as one part in a million can produce detectable physiological effects, the sudden appearance of more than enough to enable skilled researchers to determine its constitution marks a gala occasion in the field of biochemistry.

As fate would have it, Szent-Györgyi, four years after his discouraging attempts to extract appreciable amounts of the vitamin from oranges, became appointed Professor of the University of Szeged. This university is situated in an area where vast quantities of Hungarian capsicum are cultivated. This is the big red capsicum which is consumed as a vegetable in Hungary. One evening his wife gave him these peppers for supper. For some reason he could not eat them and so decided to make experiments with them. He worked all that night and next morning. Results were so encouraging that the aid of the whole household was enlisted to make pure Vitamin C from the peppers before the late season frosts should cut off the supply. After three weeks of hard labor they had a full pound of the pure crystalline ascorbic acid.

The direct outcome of this was the complete elucidation of the constitution of Vitamin C and more amazing still, the achievement of its synthesis in less than a year's time. Crystallographic measurements, spectrophotometry and X-ray analysis all served the investigators in their logical deductions of the vitamin's constitution.

C.

VACUUM TUBE CHARACTERISTICS IN RELATION TO THE SELECTION OF COINCIDENT PULSES FROM COSMIC RAY COUNTERS.

BY

LEWIS FUSSELL, Jr., M.A., and THOMAS H. JOHNSON, Ph.D.

BARTOL RESEARCH FOUNDATION

Communication No. 86.

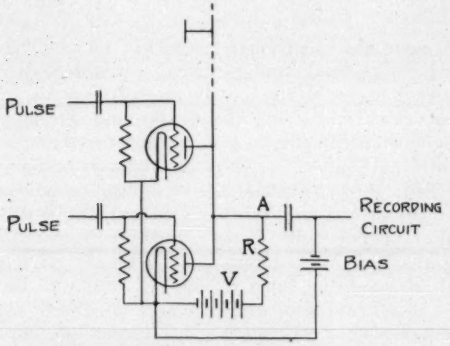
Many investigations of cosmic ray phenomena depend upon the counting of coincident discharges of two or more Geiger-Mueller counters. Although several devices have been suggested for selecting and recording coincidences, the most convenient is undoubtedly a vacuum tube circuit. The first circuit of this type, used by Bothe and Kolhorster [1] for recording coincident discharges of two counters, made use of a vacuum tube with two grids, one counter being connected to each grid. Other circuits for recording coincident pulses have also been suggested, but the most satisfactory is without doubt that proposed by Rossi [2] and incorporated into the circuit described by Johnson and Street.[3] This is represented in its essential form in Fig. 1. Each of the several counters, of which there may be any arbitrary number, is connected directly or through other stages of amplification to the control grids of a corresponding number of vacuum tubes, so that negative pulses from the counters raise the internal impedances of the tubes from low to high values. The plates of the several tubes are connected in parallel through the resistance R to the supply voltage, V, and the change in the potential of the point A constitutes the output pulse to the recording circuit. If this pulse is large enough to overcome a certain negative bias on the grid of the first tube of this circuit a count is recorded, whereas a smaller output pulse has no effect. The condition for faithful recording of coincidences is that the smallest output pulse resulting

[1] W. Bothe and W. Kolhörster, *Zs. für Phys.*, **56**, 751, 1929.
[2] B. Rossi, *Nature*, **125**, 636, 1930.
[3] Thomas H. Johnson and J. C. Street, JOUR. FRANK. INST., **215**, 239, 1933.

from discharges in all of the counters shall be greater than the largest output pulse resulting from discharges in but part of the counters.

The magnitude of the output pulse can be determined from an examination of the vacuum tube characteristics, which are shown in Fig. 2 for typical triodes. The full curve I gives the variation of current with plate potential for a single tube with the grid at its normal potential, and curve II represents the characteristic with the grid at the potential which we suppose to exist at the time of a typical input pulse.

FIG. 1.



Schematic diagram of a coincidence selecting circuit.

By means of a simple geometrical construction the output pulse can be fully investigated. The straight line of slope $-1/R$ is the characteristic on this diagram for the resistance R. For the case of a single counter and tube, the normal potential of the point A (Fig. 1) is represented by a in Fig. 2, and is determined by the intersection of the tube and resistance characteristics. A discharge of the counter changes the potential of the point A to b, and the output pulse to the recording circuit is the difference between the abscissæ a and b.

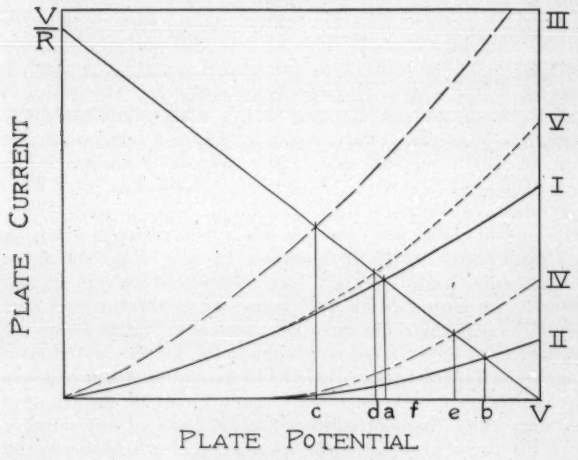
To investigate the operation of the combination of any number of tubes in parallel it is necessary to examine the combination characteristic under the normal and pulse con-

ditions. For the case of two tubes, the dashed curve III represents the combination characteristic with both grids at the normal potential; it is merely the sum of the characteristics of two individual tubes, since the current through the two is the sum of the individual currents. The dot-dash curve IV represents the two tubes when simultaneously affected by a pulse, and the dotted curve V represents the case in which one tube is affected and one is normal. The potential of A is given by c for normal conditions, by d for a "partial" coincidence, and by e for a "total" coincidence. The output pulse due to a "partial" coincidence is given by c–d, and for a "total" coincidence by c–e. The maximum output pulse for a "partial" coincidence is given by c–a, and in order to eliminate all such pulses from the recorder, the negative selecting bias must be adjusted to some value c–f greater than c–a. Total coincidences for which c–e is greater than c–f will then be recorded. The extension of this analysis to a greater number of counters and tubes is immediately obvious, since the combination characteristic of any number of tubes in parallel, with any specified grid potentials, is the sum of the individual characteristics at those grid potentials.

If the change in potential of the point A, corresponding to total coincident discharges of average size, greatly exceeds the maximum change in potential corresponding to partial coincidences, the negative bias can be adjusted to eliminate the latter, and few total coincidences will be missed. The difference between output pulses for total and partial coincidences may be defined as the 'discrimination' of the circuit. Besides depending upon the resistance R and the amplitude of the pulse, the discrimination varies considerably with the type of tube which is used.

The curves of Fig. 2 are representative of the triode class, which have usually been used in circuits of this type up to the present time. An analysis of various other types of tubes has shown that the characteristics of the pentodes, such as the type '57, are superior for this purpose. Figure 3 shows a typical case for two '57s. As before, the full curves I and II are the characteristics for a single tube under normal and pulse grid potentials respectively. Curves III and IV represent two tubes in parallel at normal and at pulse grid poten-

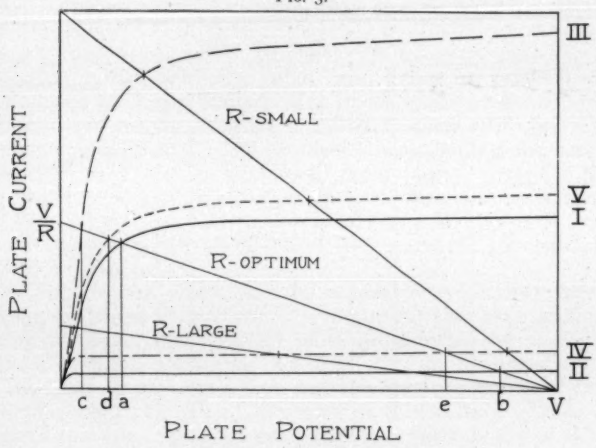
FIG. 2.



Characteristics typical of the triodes.

tials, and curve V represents the case of one tube with its grid at the pulse potential and the other at normal potential. The output pulses for partial and total coincidences are again given by *c–d* and *c–e* respectively, and their difference (the discrimination), for the value of resistance designated "*R*—optimum," is greater than it was with the triodes. In fact the comparison is even more favorable to the '57 than Figs. 2 and 3 indicate, since the input pulse chosen for Fig. 3 was actually only half that chosen for Fig. 2. The optimum value for the resistance *R* corresponds to the line which intersects just below the "knee" in the characteristic for a single tube. A smaller resistance, corresponding to "*R*—small" in Fig. 3, results in a larger output pulse for a partial coincidence without appreciably changing the total coincidence output pulse. A higher resistance, such as "*R*—large," reduces the output pulse for a total coincidence. With the optimum resistance, pulses on some but not all of the tubes can never increase the plate potential to more than a few volts, whereas coincident pulses on all the tubes, even of small size, lower

FIG. 3.



Characteristics typical of the pentodes.

the combination characteristic sufficiently to shift the potential of the point A to a high value. Table I gives partial and total output pulses from pairs of tubes of various types connected as in Fig. 1, for various sizes of input pulse. In each case the resistance R has been chosen to give maximum discrimination. The type '57 tube is better suited for coincidence selecting than any of the other tubes examined because it gives the greatest changes in the characteristic with small changes in grid potential.

For each tube the output pulse due to a partial is just half that due to a total coincidence when the input pulse is small, but as the input pulse is increased the output due to a total exceeds this value and in each case, in the limit of a "large" input pulse, the total coincidence produces a pulse equal to the normal iR drop in the resistance. Large output pulses, however, are realized in the '57 and '24-A tubes with smaller input pulses than in the triodes or power pentodes. The case of the '57 with its grid normally biased at -6 volts is particularly notable since practically full output is realized

TABLE I.

Two Vacuum Tubes in Rossi Parallel Connection.

Supply Voltage = 120 volts.

Type.	Number.	Normal Grid Bias, Volts.	Optimum Resistance, Ohms.	Input Pulse, Volts.	Output Pulses. Partial. Volts.	Output Pulses. Total. Volts.
Triodes	'01–A	0	100,000	1.5	4	9
	'27			6	6	35
				20	6	96
Tetrode	'24–A	0	25,000	1.5	1	2
				6	2	77
				15	6	115
		−6	100,000	1.5	1	2
				6	1	108
				15	2	117
Pentode	'57	0	15,000	1.5	1	3
				3	3	34
				6	10	108
				15	11	116
		−6	1,000,000	1.5	2	103
				15	2	118
Power Pentode	'89	0	5,000	1.5	0.3	0.6
				6	1	2
				15	3	20
				30	7	112

with an input pulse of only 1.5 volts. Since the pulses from a rapidly-counting counter have a wide distribution in size, and may be considerably reduced in amplitude if applied to the vacuum tube grid through a low capacity, this characteristic of the '57 is of particular practical significance. If convenience prevents the use of a bias, the '57 with its grid at the cathode potential still has a good discrimination between partial and total coincidences when the pulses are three volts or more. The tetrode '24-A is improved by a negative normal bias in regard to its discrimination for input pulses of three or more volts.

The discrimination characteristics of the triodes, on the other hand, are not sensitive to changes of normal negative grid bias, and although the operation for input pulses of six or more volts is quite satisfactory, they do not discriminate

well between partial and total coincidences when the input pulse is small. The circuit described by Johnson and Street made use of two stages of amplification before the pulses were applied to the selecting tubes, and hence all pulses were increased to 'large' values. The above considerations therefore are of no particular significance in their application to this type of circuit. The excellent discriminating characteristics of the '57 tube, with small input pulses, introduce the possibility of applying the pulses from the counters directly to the selecting circuit as shown in Fig. 1, at the same time realizing practically all of the advantages of the Johnson and Street circuit.

For circuits in which total coincidences between many counters are to be selected, it is important that the combination characteristic when all tubes are affected by an input pulse shall fall below that of a single tube in its normal state. The high amplification factor of the '57 tube makes it particularly suitable for applications of this nature. Table II gives the output pulses from a circuit containing ten of these

TABLE II.

Ten Type '57 Vacuum Tubes in Rossi Parallel Connection.

Normal control grid potential = − 6 volts.
Normal screen grid potential = 120 volts.
Input pulse applied to nine tubes for partial coincidence.

Resistance, Megohms.	Input Pulse, Volts.	Output Pulses.	
		Partial.	Total.
		Volts.	
0.2	1.5	72	103
	30	100	120
0.5	1.5	20	80
	30	45	120
1.0	1.5	0.3	55
	30	0.7	120
1.5	1.5	0.2	25
	30	0.5	120
5.0	1.5	0.1	1
	30	0.1	120

tubes connected in parallel as in Fig. 1. Under 'partial coincidences' the pulses listed are those resulting from the application of the input pulse to but nine of the tubes, the tenth remaining at its normal grid potential. Values of output pulses are given for small (1.5 volts) and large (30 volts) input pulses, and for various values of external resistance R; the optimum resistance is one megohm. Even with the small pulses, the circuit is able to discriminate well between the partial and total coincidences.

Vacuum tubes for this investigation were supplied by The R.C.A. Radiatron Co.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

TRANSPARENT CELLULOSE SHEETING FOR PRESERVING RECORDS.

The Bureau has found that the acetate type of transparent cellulose sheeting offers considerable promise as a protective covering for preserving valuable written or printed records. The high degree of transparency, tensile strength, and smoothness of cellulose sheetings are attractive features for this use. The cellulose acetate sheeting appears to be particularly well adapted since it is apparently very stable if made from high grade cellulose, can be secured in sheets only one-thousandth of an inch thick, and is thermoplastic, that is, can be applied by combined heat and pressure. The paper bearing the record is placed between two slightly larger sheets of the cellulose sheeting, and the combination is pressed between heated platens in a hydraulic press. This forms a smooth, firmly-bound unit, with edges sealed by the overlapping edges of the cellulose sheeting. The last feature is important because it makes the combination quite impervious to air.

PHASE CHANGES IN RUBBER AS INDICATED BY TEMPERATURE-VOLUME MEASUREMENTS.

The change in volume of rubber with temperature has been investigated from $-85°$ to $+85°$ C. in connection with a study of the heat capacity and related thermodynamic properties of rubber. The immediate purpose was to determine the phases in which rubber may exist, the temperatures at which transitions from one phase to another occur, and the periods of time required for transition to take place under various circumstances.

Measurements of volume change were made by enclosing samples of rubber with suitable confining liquids in dilatometers of pyrex glass and observing the changes in height of the liquid in the calibrated capillary tubes of the dilatometers.

* Communicated by the Director.

Water, alcohol, acetone, and mercury were used as confining liquids in temperature ranges for which they were respectively suitable.

The samples employed included crude rubber, purified rubber in both the sol and the gel modifications, and vulcanized compounds containing 2, 4, and 6 per cent. of combined sulphur, respectively.

The principal phase change observed was the one attendant on the well-known phenomenon of the "freezing" or raw rubber, previously studied by Bunschoten, Van Rossem, and other investigators. The same phase change is probably involved in the melting of rubber crystals as reported in the *Journal of Research* for April 1933 (RP544). The transition from the ordinary form of rubber to the more dense of "frozen" form was found to occur at about 5° to 15° C. with a decrease in volume of approximately 2.7 per cent. The transition was practically the same with the purified rubber as with the crude rubber, but no indication of any comparable change was observed in the case of the vulcanized samples.

The rate of change from the ordinary to the more dense form was found to depend on the temperature. At 0° C. about 10 days were required for the practical completion of the volume change; at somewhat lower temperatures, less time was required, but at still lower temperatures, the rate became very slow. No measurable indication of transition was observed, for example, when a sample was held below − 40° C. for three weeks.

When samples which had been cooled rapidly to below − 40° C. were warmed slowly the transition occurred with a decrease in volume which so overshadowed the effect of thermal expansion, as to give the appearance of a negative coefficient of thermal expansion. Heat capacity measurements made under similar circumstances indicated that heat was evolved as the sample was warmed. This was to be expected since the transition is accompanied by a relatively large latent heat effect.

The transition from the more dense form back to the ordinary form was not observed to take place below the range 5° to 15° C. At temperatures below this the more dense form is apparently stable.

A second type of transition, characterised by a marked change in coefficient of thermal expansion, was observed at considerably lower temperatures in the case of all samples of rubber, both vulcanized and unvulcanized.

When unvulcanized rubber was cooled without delay from room temperature to − 70° C. the coefficient of expansion decreased from approximately 0.00070 to approximately 0.00065 vol./vol./° C. Between − 70° and − 76° C., however, a change occurred lowering the coefficient to approximately 0.00025 vol./vol./° C. Rubber in the more dense form exhibited similar behavior. The coefficient was about 0.00065 vol./vol./° C. at 0° C. and decreased to about 0.00060 vol./vol./° C. in the neighborhood of −70° C., below which it dropped to approximately 0.00025. The temperatures at which the changes in coefficient of expansion occur for the two forms of rubber have not yet been determined with sufficient precision to say whether they are the same or differ slightly.

Measurements of the heat capacity of pure, unvulcanized rubber which are in progress show an anomaly in the range of − 70° to − 76° C. This probably corresponds to the change in the coefficient of thermal expansion.

Measurements of electrical properties which were made at temperature intervals of about 5° C. in connection with another investigation, gave no indication of anomaly, but when the measurements were repeated with precision at intervals of one degree, a small but definite change in dielectric constant was observed at 73° to 74° C.

It would seem reasonable also to associate this phase change with the hardening and brittleness which is observed when rubber is cooled quickly to low temperatures. However, investigators who have studied the latter phenomenon find that it occurs in the case of unvulcanized rubber at about − 55° C., which is not in accord with the present findings.

The vulcanized rubber samples showed changes in the coefficient of expansion similar to those for unvulcanized rubber at somewhat higher temperatures. The compound containing 2 per cent. of sulphur showed the change at about − 72° C.; for 4 per cent. of sulphur, the change occurred at about − 62° C., and for 6 per cent., at about −53° C.

JOINT MEETING OF INSTITUTE OF RADIO ENGINEERS AND AMERICAN SECTION, INTERNATIONAL SCIENTIFIC RADIO UNION.

In the Technical News Bulletin no. 202 (February 1934) notice was given of the arrangements made for a joint meeting of the Institute of Radio Engineers and the American Section of the International Scientific Radio Union in Washington on April 27. A preliminary program has now been arranged, consisting of two sessions at 10 A.M. and 2.00 P.M., at the building of the National Academy of Sciences, 2101 Constitution Avenue. The following papers are scheduled:

The development and characteristics of 9 cm. radiation. C. R. Kilgore (Westinghouse Company).

Vacuum tubes for generating frequencies above one hundred megacycles. C. E. Fay and A. L. Samuel (Bell Telephone Laboratories).

Facsimile radio observations during the 1932 eclipse. E. F. W. Alexanderson (General Electric Company).

Notes on propagation at a wave-length of 73 centimeters. B. Trevor and R. W. George (Radio Corporation of America).

Some recent work on the ionosphere in Canada. J. T. Henderson (Canadian National Research Council).

Studies of the ionosphere by multifrequency automatic recording. T. R. Gilliland (Bureau of Standards).

Ionosphere measurements at low altitudes. L. V. Berkner and H. W. Wells (Carnegie Institution of Washington).

High frequency ammeter. H. M. Turner (Yale University).

The thermal method of measuring the losses in a vacuum tube. F. P. Cowan (Harvard University).

Frequency standard and monitor stations of Canadian Radio Commissions. Col. W. A. Steel (Canadian National Research Council).

A method of measuring noise levels on short wave telegraph circuits. H. O. Peterson (Radio Corporation of America).

Relative daytime intensities of atmospherics. K. A. Norton (Bureau of Standards).

Developments in automatic sensitivity control. G. E. Pray (Signal Corps Laboratories).

Phase angle of vacuum tube transconductance at very high frequencies. F. B. Llewellyn (Bell Telephone Laboratories).

A new method of obtaining the operating characteristics of power oscillators. E. L. Chaffee and C. N. Kimball (Harvard University).

A short-cut method for calculation of harmonic distortion of modulated radio waves. I. E. Mouromtseff and H. N. Kozanowski (Westinghouse Company).

Space-charge effects in piezo-electric resonators. W. G. Cady (Wesleyan University).

TENSILE TESTS OF SOLDERED SPECIMENS.

Tensile tests were recently completed at room temperature and at 400° F. on specimens butt-soldered with a recently developed brazing alloy containing silver which is claimed to be particularly adapted for hard soldering of copper, brass, bronze and other non-ferrous alloys not damaged by temperatures of 1300° F., the flow point of the solder. At 400° F. specimens of copper soldered to copper broke some distance away from the joint. In all of the other tests, the fractures occurred partly through the solder, partly between the solder and the basis metal and partly in the basis metal, and indicated that the butt-soldered joints were approximately as strong as the basis metal (copper or brass) adjacent to the joint.

SIMPLIFICATION OF CUPOLA REFRACTORIES.

Simplified Practice Recommendation R154–34, Cupola Refractories, has been accorded the required degree of written approval by the industry, and is to become effective March 15, 1934. This recommendation which was proposed and developed by the industry covers the sizes and shapes of cupola lining blocks, tap-out, and slag-hole blocks. The recommendation represents a substantial reduction in the variety heretofore listed for regular stock purposes, especially for tap-out and slag-hole blocks.

Card Catalogue of North American Fossils.—According to B. F. HOWELL (*Bull. Wagner Free Inst. of Science*, 1934, IX, 105–108), "it is practically impossible for most paleontologists to consult all the museum specimens or the literature that they *ought* to consult in order to do their work properly." This is due to the rareness or inaccessibility of specimens or publications. In order to overcome this difficulty, the Wagner Free Institute of Science is about to publish the requisite information concerning North American fossils on cards. The first publication is an "Illustrated Catalogue of the Devonian Invertebrates of North America." Each card will measure 8.5 by 11 inches, and contain one or more figures of the original type specimen or specimens, a description of the species, and information concerning its known time and geographic ranges, and the place of preservation of the original type specimens. Bibliographical indexes are being prepared to accompany the cards.

J. S. H.

Eaton and Greene: Teachers of Applied Science.—PALMER C. RICKETTS (*Rensselaer Polytechnic Institute Eng. and Sci. Ser.*, 1933, XLV, 7–23) gives an account of the life and work of Amos Eaton (1776–1842) who was the first head of the faculty of the Rensselaer School, now the Rensselaer Polytechnic Institute, established at Troy, New York, in 1824 by Stephen Van Rensselaer for instruction "in the application of science to the common purposes of life." Eaton graduated at Williams College in 1799, was admitted to the bar, but devoted his life to research and teaching of the sciences, including botany, zoology, mineralogy, geology, agriculture, chemistry, and engineering. In 1824 he became professor of chemistry and natural philosophy and lecturer on geology and land surveying in the Rensselaer School, and served that institution until his death. Stress was placed upon laboratory work by each student in chemistry, analytical chemistry, physics, and natural history. Among Eaton's students were James Hall, who became state geologist of New York, James Curtis Booth, who became an eminent Philadelphia chemist, and B. Franklin Greene.

Ricketts (*Ibid.*, 26–32) also gives an account of the work of Greene, who graduated from Rensselaer in 1842, and served as its director and as a member of its faculty from 1847 to 1858. He developed the idea of a great polytechnic institute, and also stressed the value of research. He afterward became a professor of mathematics in the United States Navy.

J. S. H.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, MARCH 21, 1934.

The meeting was called to order by the President of the Institute, Mr. Nathan Hayward, who called upon the Secretary for a report of the business of the evening.

The Secretary reported that the minutes of the February meeting had been published in full in the JOURNAL OF THE INSTITUTE for March and requested that unless there were objections the minutes be declared approved as printed.

The President made such declaration.

The Secretary announced that there was no further business to be presented.

The President introduced as the speaker of the evening Dr. Hugh Stott Taylor, Chairman of the Department of Chemistry, Princeton University, who spoke on the subject "Heavy Hydrogen—A New Research Tool." Professor Taylor's paper reviewed the modern developments in connection with heavy water and particularly described the brilliant research work which has been carried out in this field in the Frick Laboratory in Princeton University. The lecture of the evening was followed by a brisk discussion of the subject and by an informal discussion which was greatly prolonged. The meeting adjourned with a formal vote of thanks to Dr. Taylor for his paper.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(*Abstract of Proceedings of Stated Meeting held Wednesday, March 14, 1934.*)

HALL OF THE COMMITTEE,
PHILADELPHIA, MARCH 14, 1934.

MR. CHARLES D. GALLOWAY *in the Chair.*

The following reports were presented for final action:

No. 2979: Improved Metal Floor Construction.

This report recommended the award of the Certificate of Merit to Mr. Joseph G. Shryock, of Philadelphia, Pennsylvania, "In consideration of the ingenious combination of standard, structural shapes in the formation of a light bridge-floor."

No. 2981: Literature.

This report recommended the award of the Louis Edward Levy Medal to Dr. Kenneth T. Bainbridge, Research Fellow of the Bartol Foundation, for his paper on "The Masses of Atoms and the Structure of Atomic Nuclei," published in the Journal of the Institute for May, 1933.

The following reports were presented for first reading:

No. 2971: Centrifuge-Microscope.

No. 2973: Atomic Hydrogen in Arc Welding.

No. 2980: Work of Mr. Ernst Georg Fischer.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTION TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, March 21, 1934.*)

RESIDENT.

MR. CHRISTIAN M. GOTTSCHAU, Mechanical Engineer, The Franklin Institute, Philadelphia, Pa.

MR. REYNOLDS D. BROWN, JR., Patent Engineer, Philadelphia Storage Battery Company, for mail: 8208 Seminole Avenue, Chestnut Hill, Philadelphia, Pa.

MR. HARRY W. KORFHAGE, 900 Maple Avenue, Collingswood, N. J.

MR. SIDNEY THOMPSON MACKENZIE, Engineering Contracting, Mackenzie Engineering Company, Philadelphia, Pa., for mail: Blue Bell, Montgomery County, Pa.

MR. ARTHUR H. NELLEN, Chief Chemist, Lee Tire and Rubber Company, Conshohocken, Pa.

MR. J. EDGAR PEW, Executive, Sun Oil Company, Philadelphia, Pa.

MR. LOUIS S. POTSDAMER, Consulting Chemical Engineer, Industrial By-Products and Research Corporation, Philadelphia, Pa.

MR. M. B. TARK, Engineer in Charge, Sanitary Division, Link-Belt Company, Philadelphia, Pa.

MR. WILLIAM H. WOOD, Engineering Assistant, Bell Telephone Company of Penna., Philadelphia, Pa.

NON-RESIDENT.

MR. G. ELLIS FLACK, Librarian, University College, Nottingham, England.

DR. JOHN W. MAUCHLY, Professor Physics, Ursinus College, Collegeville, Pa.

MR. WILLIAM S. PETTIT, Instructor, Ursinus College, Collegeville, Pa.

STUDENT.

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MR. A. ROY ROBSON, Landsdowne, Pa.
MR. CLEMENT B. WOOD, Conshohocken, Pa.

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MISS MARY ROBERTS COLES, 2010 Delancy Street, Philadelphia, Pa.
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MRS. THERON I. CRANE, The Bellevue-Stratford Hotel, Philadelphia, Pa.
MRS. CHARLES DAY, St. Georges Road, Mt. Airy, Philadelphia, Pa.
MR. A. M. E. DUNLAP, Villanova, Pa.
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MISS ROSENA FELS, Garden Court Apartments, Forty-seventh and Pine Streets, Philadelphia, Pa.
MRS. WILLIAM W. FITTER, SR., Villanova, Pa.
MRS. WILLIAM H. GREENE, 2128 Spruce Street, Philadelphia, Pa.
MR. J. B. HAMILTON, 4915 Hazel Avenue, Philadelphia, Pa.
MR. GEORGE H. HOUSTON, 123 South Broad, Street, Philadelphia, Pa.
MRS. J. BERTRAM LIPPINCOTT, 1712 Spruce Street, Philadelphia, Pa.
MISS SELINA B. MCILHENNY, The Bellevue-Stratford Hotel, Philadelphia, Pa.
MR. W. D. MASON, Sun Oil Company, Marcus Hook, Pa.
MR. NORMAN R. MCLURE, Strafford, Pa.
MR. GEORGE MYERS, Ardmore, Pa.
MR. J. H. NEESON, 1103 City Hall Annex, Philadelphia, Pa.
MR. LEONARD ORMEROD, 842 Marlyn Road, Philadelphia, Pa.
MISS LOUISA RAWLE, "Wayside" Bryn Mawr, Pa.
MISS ANNE M. REED, 2016 Spruce Street, Philadelphia, Pa.
MR. JOHN M. RUSBY, 59 Booth Lane, Haverford, Pa.
MRS. EDGAR V. SEELER, Newtown Square, Pa.
DR. JOHN S. SHARPE, Haverford, Pa.
MR. W. H. SMEDLEY, Church and Tacony Streets, Frandford, Pa.
MRS. C. MORTON SMITH, 1718 Locust Street, Philadelphia, Pa.
MR. LEWIS M. STEVENS, 7922 Lincoln Drive, Chestnut Hill, Pa.
MR. PAUL SYNNESTVEDT, 1015 Chestnut Street, Philadelphia, Pa.
MISS OLGA E. TAFEL, Overbrook, Pa.
MR. EDWARD Y. TOWNSEND, 813 Montgomery Avenue, Bryn Mawr, Pa.
MR. J. SANDERSON TRUMP, 401 Walnut Street, Philadelphia, Pa.

MR. LOUIS W. WHEELOCK, Swarthmore, Pa.
MISS MARION B. WOOD, Conshohocken, Pa.

CHANGES OF ADDRESS.

MR. F. W. BELTZ, 525 Ninth Avenue, Prospect Park, Pa.
MR. JOHN MARSHALL, c/o F. & F. Department, E. I. Du Pont de Nemours & Co., Wilmington, Del.
MR. E. M. WOODS, Box 43, U. S. Geological Survey, Enola, Pa.

NECROLOGY.

Lieutenant James W. Cottrell, Hammonton, N. J.
Mr. Henry H. Murray, Riverton, N. J.
Col. Augustus Trowbridge, Princeton, N. J.

LIBRARY NOTES.

RECENT ADDITIONS.

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BOOK REVIEWS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 463, The N. A. C. A. High-Speed Wind Tunnel and Tests of Six Propeller Sections, by John Stack, 24 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price ten cents.

This report gives a description of the high-speed wind tunnel of the National Advisory Committee for Aeronautics. The operation of the tunnel is also described and the method of presenting the data is given. An account of an investigation of the aerodynamic properties of six propeller sections is included.

The tunnel is operated on the induction-jet principle. Compressed air discharged through an annular nozzle surrounding the tunnel downstream from the test section induces a flow of air from the atmosphere through the test section of

the tunnel where the models are placed. The forces on the model are measured by a 3-component photo-recording balance.

The test results included herein comprise measurements of the lift, drag, and pitching moments of six airfoils. The sections chosen for tests have thickness ratios of 0.06, 0.08, and 0.10; three are based on the Clark Y profile and three on the R.A.F. 6 profile. The tests were made over a wide speed range and for several angles of attack, varying from that of zero lift to 12°, in order to investigate the effects of compressibility on the airfoil characteristics.

Report No. 466, Aircraft Power-Plant Instruments, by Harcourt Sontag and W. G. Brombacher, 57 pages, tables, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price fifteen cents.

This report supersedes that on aircraft power-plant instruments, published in 1921 as N. A. C. A. Technical Report No. 129, which is now, on the whole, obsolete. Aircraft power-plant instruments include tachometers, engine thermometers, pressure gages, fuel-quantity gages, fuel flow meters and indicators, and manifold pressure gages. The report includes a description of the commonly used types and some others, the underlying principle utilized in the design, and some design data. The inherent errors of the instruments, the methods of making laboratory tests, descriptions of the test apparatus, and data in considerable detail on the performance of commonly used instruments are presented. Standard instruments and, in cases where it appears to be of interest, those used as secondary standards are described. A bibliography of important articles is included.

Report No. 467, The Experimental Determination of the Moments of Inertia of Airplanes, by Hartley A. Soule and Marvel P. Miller, 15 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price five cents.

The application of the pendulum method to the experimental determination of the moments of inertia of airplanes is discussed in this report. Particular reference is made to the effects of the air, in which the airplane is immersed, on the swinging tests and to the procedure by which these effects are taken into account.

Consideration of the effects of the ambient air has shown that the virtual moment of inertia of the airplane about any given axis of oscillation must be regarded as made up of three distinct parts; namely, that of the structure, that of the air entrapped within the structure, and that of the apparent additional mass of external air influenced by the airplane's motion. As the true moment of inertia consists only of the moments of inertia of the structure and the entrapped air, the apparent additional moment of inertia due to the influence of the external air is determined and deducted from the virtual moment of inertia. The apparent additional moment of inertia is obtained by computations utilizing the results of experiments made to determine the additional-mass effect for plates of various aspect ratios.

The procedure described in this report has been used for some time, and the data on several airplanes for which the moments of inertia have been found are included. The precision is believed to be within limits of ± 2.5 percent, ± 1.3 percent, and ± 0.8 percent for the X, Y, and Z axes, respectively.

Report No. 472, Wind-Tunnel Tests on Combinations of a Wing with Fixed Auxiliary Airfoils Having Various Chords and Profiles, by Fred F. Weick and Robert Sanders, 20 pages, tables, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price ten cents.

Various auxiliary airfoils having three different airfoil sections and several different chord lengths were tested in combination with a Clark Y model wing in a sufficient number of relative positions to determine the optimum with regard to certain criterions of aerodynamic performance. The airfoil sections included a symmetrical profile, one of medium camber, and a highly cambered one. The chord sizes of the auxiliary airfoils ranged from 7.5 to 25 per cent. of the chord of the main wing, and the span was equal to that of the main wing. The tests were made in the N. A. C. A. 5-foot vertical wind tunnel.

It was found that each of the auxiliary airfoil combinations tested, regardless of size or airfoil section, had, when located at its best position, substantially higher values of the maximum lift coefficient and of the ratio $C_{L\max}{}^2/C_{D\min}$ than the main wing alone. The maximum values of the lift coefficient obtained, based on the total area, were very nearly the same with all the auxiliary airfoils tested. The symmetrical airfoils gave lower values of the minimum drag coefficient and higher values of the ratio $C_{L\max}{}^2/C_{D\min}$ than the cambered auxiliary airfoils. The highest value of the ratio $C_{L\max}{}^2/C_{D\min}$ was obtained with the symmetrical auxiliary having a chord length 14.5 per cent. of the main wing chord. The positions giving the highest values of this ratio did not vary greatly for the different auxiliary airfoils tested, except for the narrowest ones, which gave higher values in lower positions.

Report No. 473, Strength Tests of Thin-Walled Duralumin Cylinders in Compression, by Eugene E. Lundquist, 20 pages, tables, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price ten cents.

This report is the second of a series presenting the results of strength tests of thin-walled duralumin cylinders and truncated cones of circular and elliptic section. It contains the results obtained from compression tests on 45 thin-walled duralumin cylinders of circular section with ends clamped to rigid bulkheads. In addition to the tests on duralumin cylinders, there are included the results of numerous tests on rubber, celluloid, steel, and brass cylinders obtained from various sources.

The results of all tests are presented in nondimensional form and are discussed in connection with existing theory. In the theoretical discussion, it is shown that the walls of a thin-walled cylinder in compression can be correlated with the buckling of flat plates under edge compression in an elastic medium, and that perhaps many solutions for problems in the buckling of plates can, with the proper factors, be applied to similar problems in the buckling of cylinders and curved sheets.

Report No. 475, Wing Pressure Distribution and Rotor-Blade Motion of an Autogiro as Determined in Flight, by John B. Wheatley, 11 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price five cents.

This report presents the results of tests in which the pressure distribution over the fixed wing of an autogiro was determined in both steady and accelerated flight. In the steady-flight condition, the rotor-blade motion was also measured. These data show that in steady flight the rotor speed as a function of the air speed is largely affected by the variation of the division of load between the rotor and the wing; as the load on the wing increases, the rotor speed decreases. In steady flight the presence of the slipstream increased both the wing lift at a given air speed and the maximum lift coefficient of the wing above the corresponding values without the slipstream. In abrupt high-speed turns, the wing attained a normal-force coefficient of unity at almost the initial value of the air speed and experienced its maximum load before maximum acceleration occurred.

Report No. 476, Relation of Hydrogen and Methane to Carbon Monoxide in Exhaust Gases from Internal-Combustion Engines, by Harold C. Gerrish and Arthur M. Tessmann, 11 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price five cents.

The relation of hydrogen and methane to carbon monoxide in the exhaust gases from internal-combustion engines operating on standard-grade aviation gasoline, fighting-grade aviation gasoline, hydrogenated safety fuel, "Laboratory Diesel" fuel, and "Auto Diesel" fuel was determined by analysis of the exhaust gases. Two liquid-cooled single-cylinder spark-ignition, one 9-cylinder radial air-cooled spark-ignition, and two liquid-cooled single cylinder compression-ignition engines were used.

The results of more than 100 exhaust-gas analyses showed that a linear relation existed between the carbon monoxide and the hydrogen found in the exhaust gas from engines using hydrocarbon fuels. A small amount of CH_4 was found to be always present in the exhaust gas, but the amount was independent of the air-fuel ratio and of the H–C ratio of the fuel. These relationships and the use of the Ostwald combustion diagram make available all the information of a complete exhaust-gas analysis when any two factors (CO_2, CO, O_2, or air-fuel ratio) are known. The preparation and use of an Ostwald combustion diagram are described. It is also shown that the air-fuel ratio supplied to the engine may be determined with a precision of ± 2 per cent. without measuring the air taken in by the engine, thus making the method of particular value for work outside the laboratory.

Report No. 479, Stability of Thin-Walled Tubes Under Torsion, by L. H. Donnell, 24 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price ten cents.

In this paper a theoretical solution is developed for the torsion on a round thin-walled tube for which the walls become unstable. The results of this theory are given by a few simple formulas and curves which cover all cases. The differential equations of equilibrium are derived in a simpler form than previously found, it being shown that many items can be neglected. The soluton obtained

is "exact" for the two extreme cases when the diameter-length ratio is zero and infinite, and is a good approximation for intermediate cases. The theory is compared with all available experiments, including about 50 tests made by the author. The experimental-failure torque is always smaller than the theoretical-buckling torque, averaging about 75 per cent. of it, with a minimum of 60 per cent. As the form of the deflection checks closely with that predicted by theory and the experiments cover a great range of shapes and materials, this discrepancy can reasonably be ascribed largely to initial eccentricities in actual tubes.

THIS MECHANICAL WORLD, AN INTRODUCTION TO POPULAR PHYSICS, by Morton Mott-Smith, Ph.D., 232 pages, illustrations, 13 × 19.5 cms. New York and London, D. Appleton and Company, 1931. Price $2.00.

In his Introduction, the author builds an excellent case for the reading of his book by the non-scientifically trained person. He points out that the sum total of scientific knowledge and technical training is held by a relatively small number of individuals. Probably not more than one person in every thousand understands the workings of the many complicated devices that have earned for this present phase of civilization the title: "Age of Science." In the less enlightened ages it was not unusual to put to death those who dared to penetrate the veil of superstition and mystery which surrounded most natural phenomena. There seems little danger of such atavistic practices being revived today. The benefits derived from the inventions of the last two centuries are obvious to the majority and now, whenever a nation sets up a new regime the scientific mind is recognized as an indispensible factor in the new order of things.

Nevertheless, the reviewer agrees with the author, that everybody should try to gain a better understanding of this mechanical world into which we have been brought through no choice of our own. Physical Science long ago outgrew its swaddling clothes and appears concerned principally with the development of a universal mathematical expression that shall define the behavior of the ultimate in material particles. A clear case of finding out more and more about less and less.

"This Mechanical World" confines itself principally to the physics of mechanics. The author very entertainingly describes the history of the discovery and working out of these mechanical principles. Archimedes, Galileo, Newton and others receive credit due them for the part they played. This method of expanding the subject, by starting at the beginning and following the gradual development throughout the decades, is to be highly recommended. If one can gain a picture of how the best minds of that time struggled to prove some physical law which many now look upon as perfectly obvious and self-evident, then is the interest aroused and the principles more easily grasped. In addition to following a recognized pedagogic method of presentation, the author has not failed to inject bits of epigrammatic whimsicality wherever they might serve to impress some fact upon the reader's mind.

The subjects discussed include density, multiplication of force, Galileo's dynamic discoveries, the law of inertia, acceleration, conservation of momentum, gravitation, action and reaction, kinetic and potential energy. The volume is much enhanced through the addition of a chronologically arranged list of historical writings on the most important developments in macrocosmic physics.

T. K. CLEVELAND.

PUBLICATIONS RECEIVED.

Richard Trevithick, The Engineer and the Man, by H. W. Dickinson and Arthur Titley, 290 pages, illustrations, 16 × 25 cms. Cambridge, University Press, 1934. Price $5.00.

Signals and Speech in Electrical Communication, by John Mills, 281 pages, 13 × 19 cms. New York, Harcourt, Brace and Company. Price $2.00.

Thermochimie, par W. Swietoslawski, 379 pages, tables, 16 × 24 cms. Paris, Librairie Felix Alcan, 1933.

Physical Constants, Selected for Students, by W. H. J. Childs, 77 pages, tables, 11 × 17 cms. London, Methuen & Co., New York, E. P. Dutton & Co. Price $1.20.

Actualites Scientifiques et Industrielles, No. 62, Les Rayons α β γ des corps radioactifs en relation avec la structure nucleaire, par Madame Pierre Curie, 40 pages, 16.5 × 25 cms. Price 12 francs. No. 65, Sur l'Excedent de Puissance des Oiseaux, par A. Magnan et A. Planiol, 25 pages, illustrations, 16.5 × 25 cms. Price 8 francs. No. 66, Sur l'Excedent de Puissance des Insectes, par A. Magnan et A. Planiol, 26 pages, illustrations, 16.5 × 25 cms. Price 8 francs. No. 71, La Methode du Champ Self-Consistent, par L. Brillouin, 46 pages, 16.5 × 25 cms. Price 12 francs. No. 74, Spectres Moleculaires, Etude des molecules diatomiques, par P. Swings, 51 pages, illustrations, 16.5 × 25 cms. Price 14 francs. No. 77, Questions non Resolues de Geometrie Algebrique, par Lucien Godeaux, 24 pages, 16.5 × 25 cms. Price 8 francs. No. 79, Les Espaces de Finsler, par E. Cartan, 40 pages, 16.5 × 25 cms. Paris, Hermann & Cie., 1934.

Canadian Dominion Bureau of Statistics, Statistics of Dairy Factories, 1932, 96 pages, tables, 16.5 × 24.5 cms. Ottawa, King's Printer, 1934.

National Advisory Committee for Aeronautics, Technical Notes, No. 489, Aerodynamic Characteristics of Anemometer Cups, by M. J. Brevoort and U. T. Joyner, 7 pages, illustrations, 20 × 26 cms. Washington, Committee, 1934. No. 490, Tank Tests of Auxiliary Vanes as a Substitute for Planing Area, by John B. Parkinson, 7 pages, illustrations, 20 × 26 cms. Washington, Committee, 1934. No. 491, Tank Tests of a Family of Flying-Boat Hulls, by James M. Shoemaker and John B. Parkinson, 13 pages, illustrations, 20 × 26 cms. Washington, Committee, 1934. No. 492, The Aerodynamic Analysis of the Gyroplane Rotating-Wing System, by John B. Wheatley, 22 pages, illustrations, 20 × 26 cms. Washington, Committee, 1934. No. 493, Aerodynamic Rolling and Yawing Moments Produced by Floating Wing-Tip Ailerons, as Measured by the Spinning Balance, by Millard J. Bamber, 8 pages, illustrations, 20 × 26 cms. Washington, Committee, 1934. Nineteenth Annual Report, 1933, 35 pages, 23 × 29 cms. Washington, Government Printing Office, 1934. Price ten cents.

CURRENT TOPICS.

Pigment of Tomatoes.—M. B. MATTACK AND CHARLES E. SANDO (*Jour. Biol. Chem.*, 1934, CIV, 407–414) have isolated the characteristic red coloring matter of American red and purple tomatoes, and identified it as lycopene, a complex hydrocarbon which has the empirical formula $C_{40}H_{56}$. The same pigment was found in three American varieties of tomatoes, and also in an Italian variety.

J. S. H.

Occurrence of Selenium in Poisonous Plants.—The work of O. A. BEATH, J. H. DRAIZE, H. F. EPPSON, C. S. GILBERT, AND O. C. MCCREARY (*Jour. Am. Pharm. Asso.*, 1934, XXIII, 94–97) indicates that the toxicity of certain range plants to live stock is due in part, at least, to the presence of selenium compounds in these plants. The toxicity of a given species of plant may vary with the geological formation upon which it is growing; the selenium is obtained from the soil, is more abundant in the above-ground portion of a plant than in its root system, and may be only a trace or as much as 0.1 per cent. of the air-dried plant. Typical members of this group of poisonous plants are the woody aster (*Xylorhiza Parryi*), and the two-grooved milk vetch (*Astragalus bisulcatus*). All species of plants do not withdraw selenium from a soil, in which it is present, and incorporate it in their tissues. On the range, chronic selenium poisoning is designated as "blind staggers." Among its symptoms are an irritated digestive tract, a failing heart and circulation, paralysis, and, at times, impairment of vision; death is due to respiratory failure. An acutely poisoned animal may not exhibit marked pathology. In its chronic and acute forms, this type of poisoning causes heavy annual losses of live stock in the state of Wyoming.

J. S. H.

Thallium Antidotes.—JAMES C. MUNCH (*Jour. Am. Pharm. Asso.*, 1934, XXIII, 91–94) describes a definite method for combating thallotoxicosis or poisoning by salts of thallium. This procedure was developed by experiments on animals, and was successful in the treatment of 8 human beings who were poisoned by thallium. Sodium iodide is administered to render absorbed thallium insoluble

and cause its storage as its iodide. Sodium thiosulphate is then administered to control the mobilization and excretion of the stored thallium. Thallium poisoning gives rise to marked disturbances of calcium metabolism, general depression of the endocrines or ductless glands, and hypochlorhydria or achlorhydria, *i.e.*, the secretion of hydrochloric acid into the gastric juice either decreases or ceases. Administration of calcium salts, in conjunction with sodium iodide and thiosulphate, apparently favors more rapid recovery. Pilocarpine is also administered, primarily as an endocrine stimulant; it also aids elimination of the poison by stimulation of perspiration into which thallium is secreted. Hydrochloric acid is given by mouth to replace the depleted supply in the gastric juice.

J. S. H.

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